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Effect and tolerances of RF phase and amplitude errors in the ILC Crab Cavity

G. Burt[1], A. Dexter[1], P. Goudket[2],
1 Lancaster University, Cockcroft Institute, Lancaster, LA1 4YR, UK
2 ASTeC, Daresbury, Warrington, WA4 4AD, UK

Abstract

A large crossing angle (~ 20 mrad) for electron and positron bunches through the beam delivery system of International Linear Collider (ILC) is currently a favoured option of the Global Design Effort (GDE). For crossing angles greater than about 2 mrad, matching crab cavity systems are required on the electron and positron beam lines to impart an angular kick to the bunches so that they become aligned at the interaction point (I.P.). The avoidance of differential transverse deflection of electron and positron bunches at the I.P. requires almost perfect matching of the RF phase of the crab cavity system on the positron beam line to the crab cavity system on the electron beam line. In this paper the effects of phase and amplitude errors in the ILC crab cavity have been calculated, and these have been used to estimate the tolerances on the crab system.

1 Introduction

The most flexible designs of the international linear collider (ILC) beam delivery system in terms of operating parameters typically have a crossing angle between the electron and positron beamlines greater than about 14 mrad so that electron and positron bunches do not pass through each other's final focusing quadrupole doublets.

At the ILC interaction point (IP) the bunch length will be of the order of 600 times the bunch width and 60,000 times the bunch height. If positron and electron bunches, aligned with their beamlines, meet with a crossing angle θ_c, the luminosity is reduced by the luminosity reduction factor S with respect to a head on collision given as

$$S = \frac{1}{\sqrt{1+\left(\frac{\sigma_z \theta_c}{2\sigma_x}\right)^2}} \qquad (1)$$

where σ_x is the bunch width and σ_z is the bunch length [1]. Equation (1) applies for Gaussian bunches. For the 20 mrad crossing angle nominal parameter set as defined in [2], the luminosity reduction factor would be 0.16 . This loss can be recovered by aligning bunches prior to collision. One method of achieving this alignment is to use a deflecting cavity in a crabbing phase to rotate bunches rather than to deflect them. A deflecting cavity is a RF cavity that uses the first dipole mode for its operation instead of the accelerating monopole mode. The dipole mode has zero longitudinal electric field along its beam axis and a large transverse magnetic field (vertical in this case) that imparts varying transverse momentum (horizontal) along the bunch.

If the phase of the RF is timed so that the centre of the bunch passes through the cavity when the magnetic field is zero then the head and tail of the bunch will experience equal and opposite Lorentz forces, causing the bunch to start rotating. To maximise the luminosity bunches should be in line at the Interaction Point (IP), and hence must have rotated by half the crossing angle when they reach the IP. It is common to refer to a deflecting cavity phased to give a rotational kick rather than a deflectional kick as a crab cavity.

2 Transverse kick provided by the crab cavity

2.1 Approximate treatment assuming a pillbox cavity

The action of a crab cavity is most simply understood with reference to a pillbox cavity. For a pillbox cavity without beam-pipes, the TM110 dipole mode has no electric field on axis and a constant magnetic field on axis. The transverse momentum kick for a relativistic particle passing through a crab cavity is given as

$$p(t_o) = \int_{t_o}^{t_1} e\,c\,B\cos(\omega t)\,dt = -\frac{e\,c\,B}{\omega}\left\{\sin(\omega t_1) - \sin(\omega t_o)\right\} \qquad (2)$$

where B is the Magnetic flux density, c is the speed of light, e is the charge of an electron, ω is the angular frequency, t_o is the entry time of the particle and t_1 is the time the particle leaves the cavity. The time t_1 is given as,

$$t_1 = t_o + \frac{d}{c} \qquad (3)$$

where d is the cavity length. The maximum deflectional kick p_{max} is obtained when $\omega t_o = -\frac{\pi}{2}$ and $\omega t_1 = \frac{\pi}{2}$. For a relativistic bunch the energy of the kick in electron volts is given as $eV = pc$ hence

$$eV_{max} = p_{max} c = -\frac{2ec^2 B}{\omega} \qquad (4)$$

V_{max} is the maximum transverse kick (as transverse kick depends on phase).
The crab rotation effect of a deflecting cavity is also determined from equation (2) by considering the relative transverse momentum kick between the front and the centre of a short bunch. For a short bunch of length $\sigma_z = c\,\delta t$ the relative momentum kick between front and centre is given as

$$\Delta p = ecB\left\{\int_{t_o}^{t_1} dt\ \cos(\omega t) - \int_{t_o+\delta t}^{t_1+\delta t} dt\ \cos(\omega t)\right\} = ecB\{\cos(\omega t_o) - \cos(\omega t_1)\}\delta t \qquad (5)$$

The maximum kick occurs when $\omega t_o = 0$ and $\omega t_1 = \pi$ hence the optimum cavity length is given by,

$$d_{optimum} = \frac{\pi c}{\omega} \qquad (6)$$

The maximum relative momentum kick (crab kick) for the front of the bunch with respect to the bunch centre and where the centre of the bunch enters the cavity at $\omega t_o = 0$ is given as

$$\Delta p_{max} = 2e\sigma_z B \qquad (7)$$

The maximum momentum kick relative to the centre can be written in terms of the maximum energy kick using (4) so that

$$\Delta p_{max} = \frac{eV_{max}\,\omega\sigma_z}{c^2} \qquad (8)$$

If we define the crab kick voltage with respect to the momentum kick between the centre and the end of the bunch we have that

$$\Delta V_{crab} = \frac{c\,\Delta p}{e} \qquad (9)$$

hence applying (8)

$$\max\{\Delta V_{crab}\} = \frac{\omega\sigma_z}{c} V_{max} \qquad (10)$$

2.2 Generalisation to a real cavities

The treatment in the previous section is for a perfect pillbox cavity with no beam-pipes. If the cavity shape is changed or beam-pipes are added the dipole mode develops a transverse electric field on axis that also contributes to the kick. It is possible to calculate the combined effect of both the electric and magnetic field using Panofsky Wenzel theorem [3] that states

$$V_\perp = -i\int (\nabla_\perp E_z)\cdot dz \qquad (11)$$

where $V_\perp$ is a transverse kick (not necessarily the maximum kick V_{max} .

As E_z = 0 for TE modes their transverse kick vanishes. Effectively the transverse kick from the electric and magnetic fields cancel for a TE mode but add for a TM mode. In real cavities there are no true TE and TM modes just hybrids, however TE-like modes deliver much smaller transverse kicks than TM-like modes.

Applying equation (11) to a cavity excited in a dipole mode such that the longitudinal electric field is zero on axis one obtains,

$$V_\perp = -i\frac{c}{\omega}\int \frac{E_z(a,\,0,\,z)\cos\left(\frac{\omega z}{c}\right)}{a}.dz$$

where a is a point shifted in the x direction from the axis and the centre of the cavity is at $z = 0$. The cosine term comes from the transit time, see (2). For a crab cavity the transverse deflecting voltage ΔV_{Crab} seen by the front of the bunch when the cavity is phased correctly is therefore given by

$$\Delta V_{Crab} = -i\frac{c}{\omega}\int \frac{E_z(a,0,z)\sin\left(\frac{\omega z}{c}+\frac{\omega\sigma_z}{c}\right)}{a}.dz$$

For a symmetric cavity this becomes

$$\Delta V_{Crab} = -i\frac{c}{\omega}\int \frac{E_z(a,0,z)\cos\left(\frac{\omega z}{c}\right)\sin\left(\frac{\omega\sigma_z}{c}\right)}{a}.dz = \sin\left(\frac{\omega\sigma_z}{c}\right)V_\perp$$

and for small bunches

$$\Delta V_{Crab} = \frac{\omega\sigma_z}{c}V_\perp \qquad (12)$$

It is apparent therefore that (12) and hence (10) are general results.

3 Bunch rotation at the interaction point (IP)

For a bunch whose centre is at the middle of the crab cavity when its magnetic field is zero, then particles either side of the centre move away from the axis in opposite directions as the bunch travels on towards the final focus quadrupole doublets. As these off axis particles pass through the final focus quadrupoles, they get a momentum kick that is in opposition to the initial crab kick. If the crab cavity is very close to the final focus then the momentum kick from the final doublet is very small and the bunch keeps rotating in the same direction up to and beyond the IP. If the crab cavity is set back from the final focus by its nominal focal length, then the kick imparted by the final doublet exactly cancels the kick from the crab cavity hence the bunch continues to and beyond the final doublet at the angle with which it arrived. When the crab cavity is set even further back, the direction of rotation is reversed in the final doublet. As the divergence of the beam arriving at the crab cavity is non-zero, the angle that a bunch takes at the IP depends on the position of the crab cavity with respect to the final focus.

The effect of the final focus can be expressed in terms of the matrix element R_{12} that determines a transverse displacement x_{ip} of a particle at the IP that is on axis at the crab cavity and has angular direction x'_c on leaving the crab cavity, i.e.

$$x_{ip} = R_{12}\,x'_c \qquad (13)$$

The angular deflection imparted by the crab cavity for a relativistic particle is given simply by

$$x'_c = \frac{\Delta p}{mc} \qquad (14)$$

hence from (13), (14) and (8) we can write

$$x_{ip} = R_{12}\frac{e}{mc^2}\frac{V_{max}\omega\sigma_z}{c} = R_{12}\frac{V_{max}\omega\sigma_z}{cE_o} \qquad (15)$$

where E_o is the energy of the beam in eV. Defining the crab angle θ_r as half the crossing angle θ_c then

$$\theta_r = \frac{x_{ip}}{\sigma_z} = R_{12}\frac{V_{max}}{E_o}\frac{\omega}{c} \qquad (16)$$

Assuming a crab angle of 10 mrad, a beam energy of 500 GeV, a cavity frequency of 3.9 GHz and R_{12} = 16.3 m rad^{-1} (horizontal) then V_{max} = 3.74 MV.

The value of R_{12} is typically determined computationally hence its dependence on the position of the crab cavity is not immediately apparent. If however we write the general transfer matrix M_f in the horizontal x plane of a non-dispersive focusing lens system of focal length f as

$$M_f = \begin{bmatrix} 1+\delta_1 & \delta_2 \\ -\frac{1+\delta_1}{f} & 1+\delta_1+\delta_3 \end{bmatrix} \qquad (17)$$

where δ_1 , δ_2 and δ_3 become zero for a thin lens, then it can be shown by considering a drift L_c from the crab cavity to front of the lens system followed by a drift L_p from the end of the lens system to the IP that R_{12} is given by

$$x_{ip} = R_{12}x'_c = \left\{(1+\delta_1)\left(L_p + L_c - \frac{L_pL_c}{f}\right) + (\delta_2 + L_p\delta_3)\right\}x'_c \qquad (18)$$

Equation (18) also gives the displacement x either side of the IP for particles that enter the crab cavity on axis. This formula shows that when $L_p > f$ then the displacement x_{ip} and hence the crab rotation decreases as the crab cavity is moved further back from the final focus doublet.

When focusing is at the IP from a transfer matrix analysis one can show that

$$(1+\delta_1)\left(1-\frac{L_p}{f}\right)(L_u - L_c) + (1+\delta_1)\left(L_p + L_c - \frac{L_pL_c}{f}\right) + (\delta_2 + L_p\delta_3) = 0$$

where L_u is the distance from the β function minimum to the front of the lens system. It follows that (18) can be written

$$x_{ip} = (1+\delta_1)\left(1-\frac{L_p}{f}\right)(L_c - L_u)x'_c \qquad (19)$$

Care should be used when interpreting this equation as when $L_p = f$ the value of L_u goes to infinity and x_{ip} stays finite. By consideration of how the β function transforms equation (19) is equivalent to

$$x_{ip} = \sqrt{\beta_{ip}}\sqrt{\beta_{crab} - \beta_o}\; x'_c \qquad (20)$$

The usefulness of this expression is that R_{12} can be determined from knowledge of the β function.

4 Transverse offset

As well as giving bunches a distributed momentum kick the crab cavity also gives the bunch a horizontal displacement. As the centre of the bunch enters the cavity it first sees a deflecting force in the horizontal plane that causes it to move off-axis. After the bunch crosses the centre of the cavity it experiences a deflecting force equal and opposite to the force seen in the first half of the cavity. This force stops the transverse motion but does not restore the bunch position back on-axis.

The displacement of the particles occurring in the crab cavity is computed as

$$x_{cavity} = \int_{t_0}^{t_1} \frac{p_T(t')}{m} dt' \qquad (21)$$

For the TM110 mode, the displacement for optimum cavity length (5) as the bunch leaves the crab cavity is given as

$$x_{cavity} = -\frac{ecB}{m\omega^2}\left(\pi\sin(\omega t_o) - 2\cos(\omega t_o)\right) = -\frac{V_{max}}{2E_o}\frac{c}{\omega}\left(\pi\sin(\omega t_o) - 2\cos(\omega t_o)\right) \qquad (22)$$

For a beam energy of 500 GeV, a cavity frequency of 3.9 GHz and V_{max} = 3.74 MV then this displacement ~ 0.09 μm

Specifying the focusing doublet by the matrix M_f of expression (16) the displacement x_{cavity} is reduced at the IP by the factor $(1+\delta_1)\left(1-\frac{L_p}{f}\right)$. As L_p is close to f the resulting displacement at the IP is an order of magnitude smaller than the displacement at the crab cavity and hence can be neglected.

5 Timing Errors

Zero displacement of the bunch centres at the IP only occurs when the bunches are perfectly synchronised with the RF field in the crab cavities such that the bunch centre is at the middle of the crab cavity when the magnetic field is zero. The anticipated jitter on the arrival time of bunches at the crab cavities from the linacs will be about 0.4 ps.

For a bunch that enters a crab cavity of optimum length at time t_o and leaves at time t_1 then from (3) and (6)

$$\omega t_1 = \omega t_o + \pi \qquad (23)$$

(Note that perfect synchronism for a crab cavity with field time dependence specified as in (2) occurs when $t_o = 0$.) For a cavity of optimum length (2) gives

$$\Delta p_{opt}(t_o) = -\frac{ecB}{\omega}\{\sin(\omega t_o + \pi) - \sin(\omega t_o)\} = \frac{2\,ecB}{\omega}\sin(\omega t_o) \qquad (24)$$

and using (4), (13) and (14) this may be written

$$\Delta x_{ip} = R_{12}\frac{V_{max}}{E_o}\sin(\omega t_o) \qquad (25)$$

using (16) this may also be written as

$$\Delta x_{ip} = c\theta_r \frac{\sin(\omega t_o)}{\omega} \approx c\,\theta_r\, t_o \qquad (26)$$

Hence for a timing error of $t_o = 0.4$ ps and when half the crossing angle $\theta_r = 0.01$ rads then the bunch offset is 1.2 μm. This is unacceptably large unless the electron crab cavity has nearly the same kick as the positron crab cavity so that electron and positron bunches have the nearly the same offsets at the i.p.

To estimate timing errors there are three characteristic cases:-

1. Beams and one cavity are in synchronism but other cavity is out of time,
2. Crab cavities and one beam are in synchronism but other beam is out of time,
3. Crab cavities are in synchronism but beams are in differing synchronism.

We shall refer to case 1 as a cavity timing error and case 2 as a beam timing error. Case 3 will be regarded as a sub case of 2 rather than 1.

5.1 Bunch displacement due to cavity timing errors

Displacement for a cavity timing error is given below by equation (27). To obtain the result replace $\cos(\omega t)$ in (2) with $\cos(\omega t + \phi)$ where ϕ is the cavity phase error and set $t_o = 0$. The steps that gave (26) now give

$$\Delta x_{ip} = c\theta_r \frac{\sin(\phi)}{\omega} \approx \frac{c\,\theta_r\,\phi}{\omega} \qquad (27)$$

If one considers the actual bunch to be part of a much longer virtual bunch there will be some part of the virtual bunch that enters the cavity at time t_o such that $\omega t_o - \phi = 0$ consequently this part travels without deflection along the beam-line. The actual bunch can therefore be considered as having been rotated about the un-deflected part. This concept only works for small timing errors as the sine term in (27) bends the virtual bunch, see figure 1.

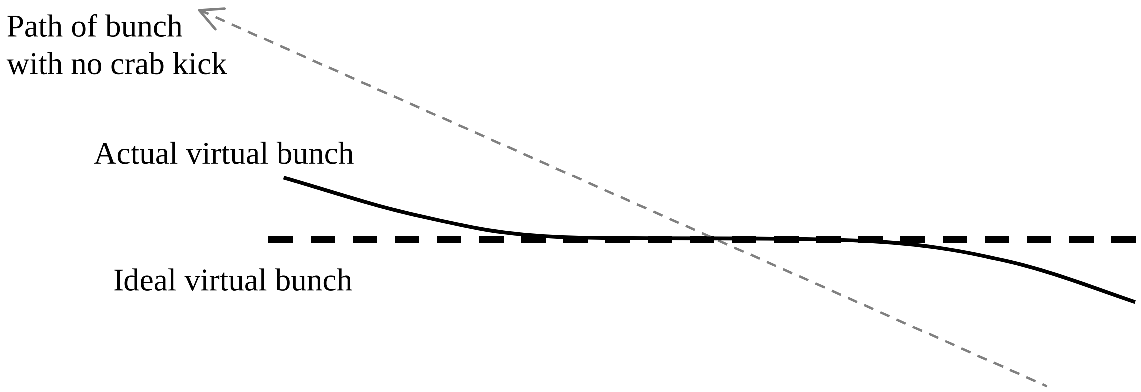

Figure 1. Shape of very long bunch rotated by a crab cavity

5.2 Geometrical luminosity loss due to bunch displacement

If at the IP the positron bunch has a horizontal displacement of $0.5\Delta x$ and the electron bunch has a displacement of $-0.5\Delta x$ and both bunches have Gaussian profiles then the integral that determines the geometric luminosity contains the term

$$f(x) = \frac{1}{2\pi\sigma_x^2}\ \exp\left[-\frac{(x+0.5\Delta x)^2}{2\sigma_x{}^2}\right]\ \exp\left[-\frac{(x-0.5\Delta x)^2}{2\sigma_x{}^2}\right] \qquad (28)$$

The luminosity reduction factor therefore given as

$$S = \exp\left(-\frac{\Delta x^2}{4\sigma_x{}^2}\right) \qquad (29)$$

Putting numbers in (29) for the 500 GeV centre of mass nominal parameter set one sees that a luminosity reduction of 2% (S = 0.98) when the horizontal beam size is **665 nm** at the IP, occurs for a displacement of 186 nm , which for a 20mrad crossing angle corresponds to a 0.029° jitter at 1.3GHz and 0.087° at 3.9 GHz. This phase error corresponds to a timing error of 0.062 ps. For the 1 TeV nominal parameter set the horizontal beam size at the IP is 554 nm hence 2% luminosity loss occurs for a displacement of 139 nm, which for a 20mrad crossing angle corresponds to a 0.065° jitter at 3.9 GHz. This phase error corresponds to a timing error of 0.047 ps. The RMS luminosity loss from phase jitter is found to be approximately equal to the single bunch luminosity loss.

5.3 Luminosity loss from bunch timing errors

If the cavities are synchronised with each other, one bunch arrives on time and the other is late (or early) then considering figure 1, the bunch that arrives on time can be regarded as a small segment of the virtual bunch that lies on the beam-line and the late (or early) bunch can be regarded as a small segment of the virtual bunch that lies off the beam-line. Effectively figure 1 and equation (26) tells us that late bunches get more kick and if the are not too late they get just enough extra kick to be in line with where they ought to be at any instant. This means that they still collide head on, but at a distance $0.5c\Delta t$ for the intended IP and with a horizontal offset of $0.5\theta_r c\Delta t$. The late (or early) bunch is only in perfect alignment with the correctly timed bunch when the latter is at the IP. Because the actual interaction occurs at a small distance from the intended IP, there will be a small amount of over/under rotation, see figure 2.

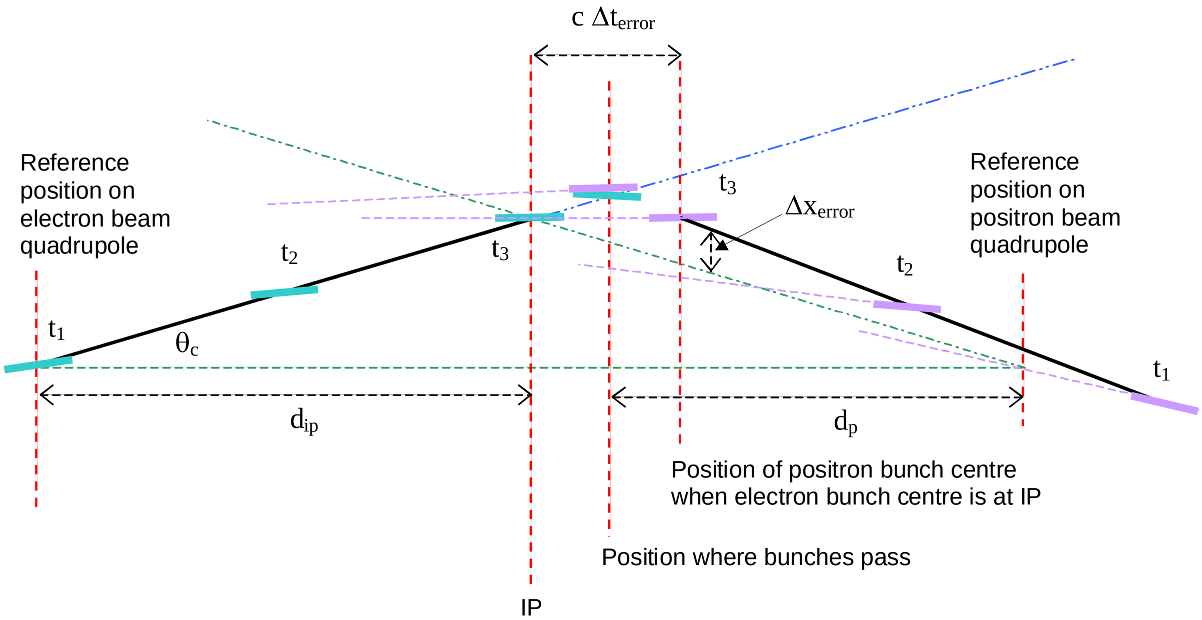

Figure 2. Graphical analysis of bunch timing errors

Explicitly the horizontal offset for bunch that is late by Δt with respect to a bunch that arrives on time is given as

$$x_{offset} = \frac{c\theta_r}{\omega}\left[\sin(\omega\Delta t) - \omega\Delta t\right] \qquad (30)$$

The luminosity reduction factor can then be found using equation (29). As before to ensure that the luminosity loss is less than 2% the offset must be less than 142 nm hence for a 20 mrad crossing angle and a crab cavity frequency of 3.9 GHz equation (30) indicates that Δt should be less than 7.7 ps. This tolerance of 7.7 ps does not include effects of bunch over rotation, incorrect rotation associated with incorrect kick, change in bunch size due to focusing (the hourglass effect) and beam to beam interaction effects. Focusing and beam to beam interactions effects do not depend on the crab cavity however it is of interest to consider the loss in luminosity resulting from moving the IP with consequential loss of focus as it is far more restrictive than beam timing issues associated with the crab cavity.

Considering the focusing effect first, size variation $\Delta\sigma$ at a distance Δz from the focus can be estimated as $\Delta\sigma \approx \sqrt{\sigma_x^2 + \frac{\varepsilon}{\beta^*}\Delta z^2} - \sigma_x$ where for the ILC nominal 1TeV parameter set the horizontal emittance $\varepsilon = 1.02\times10^{-11}$ and size at the IP $\sigma_x = 0.554\times10^{-6}\ m$ and the beta function at the ip $\beta^* = 0.03m$. A similar equation applies to vertical focusing where the vertical emittance $\varepsilon_y = 4.09\times10^{-14}$ and size at the IP $\sigma_y = 3.5\times10^{-9}\ m$ and the beta function at the ip $\beta^* = 3.0\times10^{-4}m$. For a bunch that is 7.7 ps late then the shift in the IP is given by $\Delta z = 1.16mm$ hence $\Delta\sigma_x = 0.41nm$ and $\Delta\sigma_y = 10.4nm$ The relative variation in the horizontal is tiny however the relative variation in the vertical size is huge. It is of interest therefore to estimate the luminosity reduction associated with a change in the bunches lateral dimensions. Explicitly we consider the horizontal variation in size however the resulting formula generalises to the vertical dimension.

If at the IP the positron bunch has an increased size of $+\Delta\sigma_1$ and the electron bunch has an increased size of $+\Delta\sigma_2$ and both bunches have Gaussian profiles then the integral that determines the geometric luminosity contains the term

$$f(x) = \frac{1}{2\pi(\sigma_x + \Delta\sigma_1)(\sigma_x + \Delta\sigma_2)} \exp\left[-\frac{x^2}{2(\sigma_x + \Delta\sigma_1)^2}\right] \exp\left[-\frac{x^2}{2(\sigma_x + \Delta\sigma_2)^2}\right] \quad (31)$$

Performing the x integration one can show that the luminosity reduction factor with respect to a collision at the focus is given as

$$S = \frac{1}{\sqrt{0.5\left(1+\frac{\Delta\sigma_1}{\sigma_x}\right)^2 + 0.5\left(1+\frac{\Delta\sigma_2}{\sigma_x}\right)^2}} = \frac{1}{\sqrt{1+\frac{\varepsilon}{2\beta^*\sigma_x^2}\left(\Delta z_1^2 + \Delta z_2^2\right)}} \quad (32)$$

For the current ILC beam delivery system the beam waist foci for each beam are planned to be at $\Delta f = 230\mu m$ upstream of the IP hence the luminosity reduction factor becomes

$$S = \frac{\sqrt{1+\frac{\varepsilon}{\beta^*\sigma_x^2}\Delta f^2}}{\sqrt{1+\frac{\varepsilon}{2\beta^*\sigma_x^2}\left\{(\Delta f - \Delta z)^2 + (\Delta f + \Delta z)^2\right\}}} = \frac{\sqrt{1+\frac{\varepsilon}{\beta^*\sigma_x^2}\Delta f^2}}{\sqrt{1+\frac{\varepsilon}{\beta^*\sigma_x^2}\left(\Delta f^2 + \Delta z^2\right)}} \quad (33)$$

The luminosity reduction factor arising as a consequence of focusing effects when one bunch is late as determined by (33) is plotted in figure 3. Parameters correspond to 1 TeV centre of mass.

The variation of the plot with Δf is very small. It is a apparent from figure 3 that a timing error that gives a significant luminosity reduction from vertical focusing effects is very small compared a timing error that gives luminosity loss associated with reduced kick at the crab cavity. For the ILC to achieve its luminosity target beam timing will need to be better than 0.5 ps with respect to vertical focusing and additional effects from the crab cavity are very small when this beam timing target is achieved.

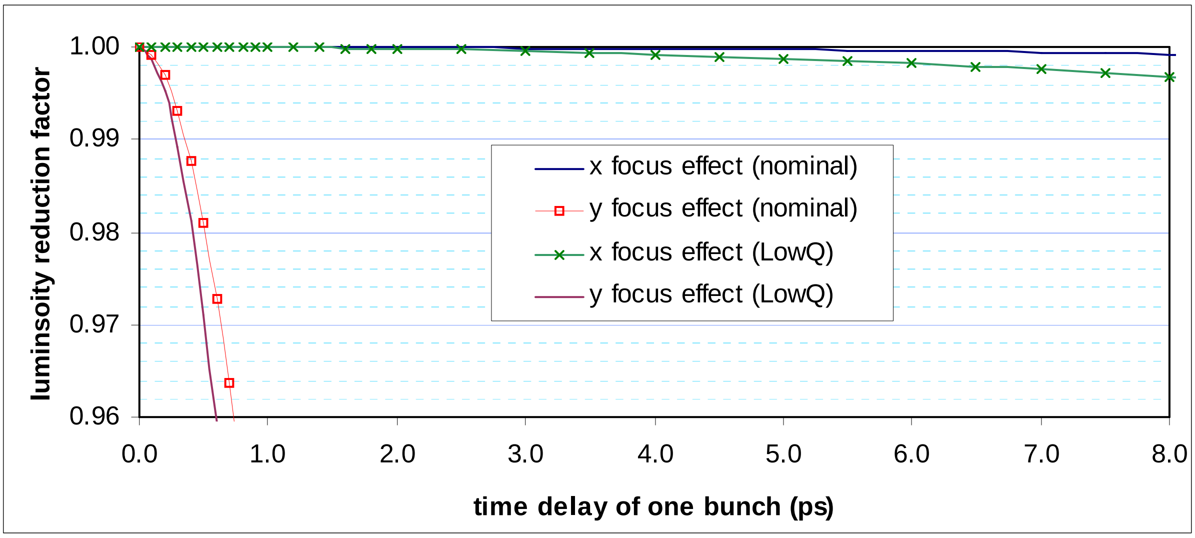

Figure 3 Luminosity reduction factor as a consequence of a late bunch shifting the interaction point considering focusing effects only.

Beam timing errors also lead to incorrect rotation. Most of the contribution to incorrect rotation comes from incorrect kick rather than additional rotation between the intended IP and the actual IP. If the crab cavity is right next to the final focusing doublet then the crab angle is set up over a distance of the order of 15 metres. In this case the bunch passes through the first focusing quadrupole with little change to its initial rate of rotation. On passing through the defocusing quadrupole the rate of rotation is significantly reduced hence the rate of rotation near the IP should be estimated on the basis that the angle of rotation was achieved over a distance of 16.3 metres for the 1TeV nominal case. For the beam timing error of 7.7 ps giving 2% luminosity loss from displacement errors, then the IP will be shift by 1.16 mm hence additional rotation will be less than half the crossing angle times this shift divided by 15 metres which equals 0.0007 mrads. From (1) it is apparent that this extra rotation will give no significant luminosity loss and indeed a timing error of 7.7 ps was intolerable from focusing effects anyway.

Looking now at the under rotation due to an incorrect kick, the rotation angle between the two bunch at collision is determined as

$$\sin(\theta) = \frac{x_{offset}(c\Delta t + \sigma_z/c) - x_{offset}(c\Delta t - \sigma_z/c)}{2\sigma_z} \qquad (34)$$

which using (30) gives

$$\sin(\theta) = \theta_r \frac{c}{\omega\sigma_z}\sin\left(\frac{\omega\sigma_z}{c}\right)\cos(\omega\Delta t) \approx \theta_r\left(1 - 0.5\omega^2\Delta t^2\right) \qquad (35)$$

The luminosity reduction factor can then be calculated using equation (1) where θ_c is replaced by $\theta_r - \theta$ i.e.

$$S = \frac{1}{\sqrt{1+\left\{\frac{\sigma_z(\theta_r - \theta)}{2\sigma_x}\right\}^2}} \approx \frac{1}{\sqrt{1+\left\{\frac{\sigma_z\,\theta_r\,\omega^2\Delta t^2}{4\sigma_x}\right\}^2}} \qquad (36)$$

For a beam timing error of 7.7 ps the loss in luminosity predicted by (36) is fractions of a percent hence this correction is also not an issue.

6 Amplitude error

From (16) one sees that the crab rotation angle of the bunch at the IP is linearly proportional to the cavity voltage, hence any variation in the voltage ΔV produces a proportional variation in the rotation of the bunch. The luminosity reduction factor S due to the voltage variation is then determined by (1) where θ_c is replaced by $2\Delta\theta$ which is determined from (16) as

$$\Delta\theta = R_{12}\frac{\Delta V}{E_o}\frac{\omega}{c} = \theta_r \frac{\Delta V}{V_{max}} \quad (37)$$

Hence the acceptable amplitude variation is given as

$$\frac{\Delta V}{V_{max}} = \frac{1}{\theta_r}\frac{\sigma_x}{\sigma_z}\sqrt{\frac{1}{S^2}-1} \quad (38)$$

Taking the luminosity reduction factor as 0.98, $\sigma_x = 0.665\times10^{-6}\,\mathrm{m}$, $\sigma_z = 0.3\times10^{-3}\,\mathrm{m}$ and $\theta_r = 0.01\,\mathrm{rads}$ then $\left.\frac{\Delta V}{V_{max}}\right|_{S=0.98} = 4.4\%$

which at a glance is a comfortably large. However this tolerance may actually be difficult to achieve if the bunches arriving at the crab cavity arrive off axis. It has been estimated that bunches could arrive at the crab cavity with a horizontal offset as a much as 1.5mm. Arriving off axis will result is a slightly differing transverse kick but more worryingly the bunch will deposit or extract energy from the cavity. This will alter the amplitude and indeed the phase of the field in the cavity for the next bunch. Detailed calculations to determine whether the tolerances established can be maintained with off-axis bunches are required. If the tolerances cannot be met, one solution might be to consider a larger cavity operating at a lower frequency.

7 Beam-beam disruption effects

Using the ILC 1 TeV nominal parameter set, recent results by Church [4] using the e^+/e^- collision simulation code “Monte Carlo” that incorporates beam to beam interaction effects and beam energy distributions, show that beam timing errors can be up to 0.67 ps before 2% luminosity is lost. This figure is in agreement with our predictions in figure 3 derived from defocusing effects alone, geometrical effects being negligible.

Simulations also performed in [4] predict a 2% luminosity loss for an amplitude jitter of 4.4% in agreement with our value derived from (35). They also predict a 2% luminosity loss for uncorrelated cavity phase timing errors of 0.043 ps, for a bunch with a horizontal width of 655nm. The uncorrelated cavity phase timing error is related to the phase errors calculated in this paper by factor of $1/\sqrt{2}$, as we calculated the phase difference between two cavities and Church calculates the error of the cavities from a perfect reference. We calculate an uncorrelated cavity phase timing error of 0.044 ps using the method detailed above. The difference could be due to beam-beam disruption effects.

Church [4] also shows that the energy tolerance on the beam needs to better than 0.3% for luminosity losses to be less than 2%. This final result is related to movement of the beam waist focus with energy.

8 Tolerances to Transverse Wakefields

The presence of wakefields in the crab cavity can have many effects on the beam, however we will concern ourselves only with the deflecting kick given to the centre of the bunch. If there is a transverse kick given to the bunch in a crab cavity it will produce an offset at the IP that will reduce the luminosity as given in (29). For the ILC 1 TeV nominal parameter set for a luminosity loss of less than 2% this offset must be less than 139 nm in the horizontal plane and 0.875 nm in the vertical plane. These offset tolerances can be used in (25) to calculate the maximum deflecting voltage tolerance at the crab cavity. Assuming a 0.5 TeV beam with R12 = 16.3 m/rad horizontal and R12 = 2.4 m/rad vertical, we calculate the maximum voltage in the horizontal plane to be 4.25 kV, and 0.182 kV in the vertical plane.

9 Conclusion

Phase and amplitude tolerances have been calculated for the ILC crab cavity system. It is shown that the system has high tolerance with respect to luminosity loss for bunch arrival time and amplitude errors, however there are tight limits on the cavity-to-cavity phase errors. To stay within the luminosity budget the positron crab cavity must have on average an identical phase to the electron crab cavity to within 0.066 degrees. This corresponds to a timing accuracy of 0.047 ps or a signal path length accuracy of 10 μm. For the 20 mrad crossing angle parameter set the crab cavities are 34 metres apart. Given that the beams will have differing offsets and hence will be delivering or extracting differing amounts of power from the cavities, the favoured option is to drive the two sets of cavities with differing power sources. Since most of the luminosity loss associated with cavity timing errors comes from associated beam deflection one might consider measuring horizontal beam deflection some distance after the crab cavity and controlling this deflection to a set point. This option is not possible unless the crab cavities are moved back from the final doublet.

	nominal 500GeV	Low Q 500GeV	Large Y 500GeV	Low P 500GeV	High Lum 500GeV
Jitter (degees)	0.0766	0.0579	0.0579	0.0529	0.0529
Amplitude %	0.0443	0.0670	0.0201	0.0459	0.0612
	nominal 1TeV	Low Q 1TeV	Large Y 1TeV	Low P 1TeV	High Lum 1TeV
Jitter (degees)	0.0648	0.0459	0.0429	0.0429	0.0374
Amplitude %	0.0375	0.0398	0.0124	0.0124	0.0325

Table 1 Phase jitter and amplitude tolerances for the suggested ILC parameters

Currently it is planned that the cavities be synchronised with a LLRF signals. The phase tolerance is on the limit of what is likely to be achievable. One might anticipate that initially the phase error will be slightly outside the tolerance dropping the luminosity by 1% or more. Use of beam position monitors or indeed the luminosity signal within the control algorithm should allow us to recover this loss if the loss cannot be recovered in other ways.

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References

[1] B. Muratori, Luminosity in the presences of offsets and a crossing angle , CERN-AB-Note-2003-026 (ABP)

[2] T. Raubenhiemer, Suggest ILC beam parameter range, http://www-project.slac.stanford.edu/ilc/acceldev/beampar/Suggested%20ILC%20Beam%20Parameter%20Space.pdf

[3] W. K. H. Panofsky and W. A. Wenzel, Some Considerations Concerning the Transverse Deflection of Charged Particle Radio-Frequency Fields, Rev. Sci. Instr. 27, 967 (1956).

[4] M. Church, Timing and amplitude tolerances for Crab Cavity, FNAL ILC docdb-295