Optimal Phasing and Energy Spread
in the ILC Main Linac

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The Basics (a revision)

The longitudinally correlated energy spread in a linac is the sum of the effects of the RF curvature and the single-bunch beam loading (longitudinal wakefield). The energy gain per unit length along the bunch can be expressed as

$$\Delta E(z) = G_0 \cos(kz + \phi) + q w_{\parallel}(z)\,, \tag{1.1}$$

where G_0 is the accelerating gradient, $k = 2\pi f_{RF} / c$, ϕ is the RF phase with respect to the bunch centre, q is the total bunch charge. $w_{\parallel}(z)$ is the bunch wakefield per unit charge given by

$$w_{\parallel}(z) = \int_{s=z}^{\infty} \rho(s) W_{\parallel}(s - z)\, ds\,, \tag{1.2}$$

where $\rho(z)$ is the normalised longitudinal bunch distribution, and $W_{\parallel}(z)$ is the point-like longitudinal wake potential.

The *loss parameter k* is the average beam loading (energy loss) per meter per unit bunch charge:

$$k_{\parallel} = \int_{-\infty}^{\infty} \rho(z) w_{\parallel}(z)\, dz\,. \tag{1.3}$$

The average enrgy gain per meter is given by

$$\left\langle \Delta E \right\rangle = \int_{-\infty}^{\infty} \rho(z) \Delta E(z)\, dz = \int_{-\infty}^{\infty} \rho(z) G_0 \cos(kz + \phi)\, dz + q k_{\parallel}\,. \tag{1.4}$$

If we assume a bunch with a Gaussian longitudinal bunch distribution with an RMS of σ_z, (1.4) becomes

$$\left\langle \Delta E \right\rangle = e^{-\frac{1}{2} k^2 \sigma_z^2} G_0 \cos(\phi) + q k_{\parallel}\,. \tag{1.5}$$

The induced relative RMS *energy spread* per unit length can be calculated in the usual fashion:

$$\sqrt{\langle \Delta E^2 \rangle - \langle \Delta E \rangle^2} = \left[\int_{-\infty}^{\infty} \rho(z) \Delta E^2(z) dz - \langle \Delta E \rangle^2 \right]^{\frac{1}{2}} \tag{1.6}$$

Since the beam loading (bunch wakefield) is always deccelerating, a minimum in the relative energy spread can be achieved by running the bunch forward of the RF crest ($\phi > 0$). The value of the optimum phase (ϕ_0) is a function of the applied RF gradient (G_0) and the bunch length (σ_z).

In this note we will investigate the optimum phase for the ILC as a function of gradient and bunch length. We will assume for all gradients[1] the form of the longitudinal wake potential given in [1]:

$$W_{\parallel}(z) = -38.1 \left[1.165 \cdot \exp\left(-\sqrt{\frac{z}{3.65\times10^{-3}\,\mathrm{m}}} \right) - 0.165 \right] \ \mathrm{V/pC/m} \tag{1.7}$$

Parameter space

For the purpose of this study, I have assumed RMS bunch lengths of 150, 200, 250 and 300 μm. For the gradients, I have taken 25, 30, 35, and 40 MV/m. I have always assumed a bunch of 2×10^{10} particles, corresponding to a charge of 3.2 nC. The results are valid for a longitudinal Gaussian distribution. Where relevant I have assume an efffective cavity length of 1.036 m.

Loss Parameters

σ_z μm	**k (V/C)**	**$\Delta V_{\mathrm{cavity}}$ (kV)**
150	1.57×10^{13}	50.2
200	1.51×10^{13}	48.4
250	1.46×10^{13}	46.9
300	1.42×10^{13}	45.6

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[1] this assumes that the basic geometry of the cavity is not changed. For high gradients (particularly the 40MV/m case), it is likely that the proposed low-loss structure will be used, and this will have a different wake potential.

Optimum Phase (minimum relative energy spread)

Optimum phase angle (in degrees)

	gradient MV/m			
σ_z μm	**25**	**30**	**35**	**40**
150	14.3	11.9	10.2	8.9
200	10.2	8.5	7.2	6.3
250	7.8	6.5	5.6	4.9
300	6.2	5.1	4.4	3.9

Resulting minimum relative energy spread $\times 10^{-4}$

	gradient MV/m			
σ_z μm	**25**	**30**	**35**	**40**
150	2.73	2.25	1.91	1.66
200	2.62	2.17	1.84	1.60
250	2.54	2.10	1.78	1.55
300	2.46	2.03	1.73	1.51

Average energy gain per cavity (MV)

	gradient MV/m			
σ_z μm	**25**	**30**	**35**	**40**
150	25.05	30.36	35.64	40.89
200	25.44	30.69	35.92	41.14
250	25.61	30.83	36.04	41.24
300	25.70	30.91	36.10	41.29

On crest operation (ϕ=0)

Resulting relative energy spread $\times 10^{-4}$

	gradient MV/m			
σ_z μm	**25**	**30**	**35**	**40**
150	10.49	8.73	7.48	6.55
200	10.01	8.33	7.14	6.24
250	9.60	7.99	6.85	5.99
300	9.24	7.69	6.59	5.76

Average energy gain per cavity (MV)

	gradient MV/m			
σ_z μm	**25**	**30**	**35**	**40**
150	25.85	31.03	36.21	41.39
200	25.85	31.03	36.21	41.39
250	25.85	31.03	36.21	41.39
300	25.85	31.03	36.21	41.39

References

[1] TESLA TDR, DESY 2001-011, March 2001.