An Introduction to Resistive Wall Wakefields

And Initial Calculations of

Their Effects for the Helical Undulator for TESLA

DUNCAN SCOTT

ID GROUP:
DARESBURY LABORATORY

ABSTRACT

The effects of stray electromagnetic fields due to a charged particle beam interacting with its environment are presented. The concepts of wake-fields and their Fourier transform, impedance, are also introduced. Formulas for an infinitely long round pipe are presented. These formulas are applied for the present, worst case geometry for the TESLA helical undulator for the production of polarised positrons.

1. INTRODUCTION

1.1 ACKNOWLEDGEMENTS

A lot of the work presented in this introduction is based on the book "*The Physics of Collective Beam Instabilities in High Energy Accelerators*" by A. Chao[1] and the lectures given by Palumbo, Vaccaro and Zobov at the 5th Advanced Accelerator Physics CERN School.[2] I am also indebted to Vasili Tsakanov, and especially Mikhail Ivanian, from the Centre for the Advancement of Natural Discoveries using Light Emission, CANDLE, in Yerevan, Armenia for their help in understanding basic concepts and the many useful discussions.

1.2 OVERVIEW

For an electron beam in free space or in a zero resistance pipe an ultra-relativistic particle (i.e. $v = c$) does not see the fields of the other particles in the beam – unless they are at the same longitudinal co-ordinate. At this co-ordinate they do see each others fields but experience no Lorentz force because $\vec{\boldsymbol{E}}$ exactly cancels $\vec{\boldsymbol{B}}$. (In the rest frame ***there is*** an electrostatic force between the particles but on performing the Lorentz transformation to the laboratory frame the motions are infinitely time dilated.) Therefore, for an instability to occur, the beam must not be ultra-relativistic or in a perfectly conducting smooth pipe. We are going to look at the latter case, where there is an ultra-relativistic beam in a beam pipe that has some resistance and is not smooth.

1.3 PERTURBATIONS OF THE ACCELERATOR MODEL

Assuming that the design orbit for an accelerator is a circle with a circumference of $2\pi R$, Figure 1, then the un-perturbed motion of a particle is modelled as a 3-D simple harmonic oscillator in the transverse and longitudinal co-ordinates. (Although it is impossible to have simultaneous 3-D focussing at a given time and space – we can consider the accelerator to be a potential well of this form in an average sense.)

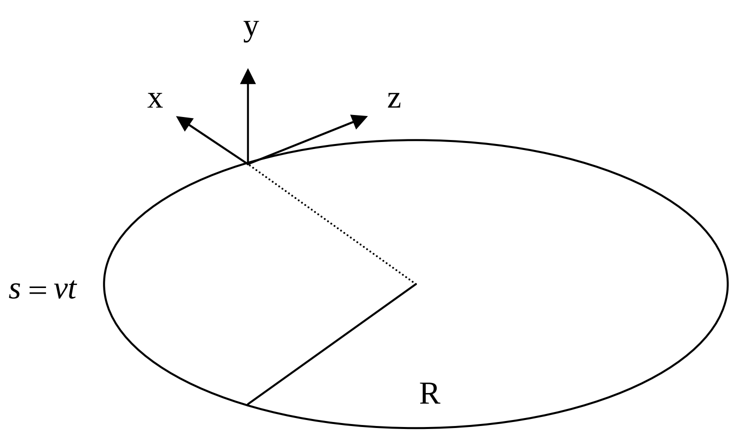

Figure 1: schematic of the accelerator model.

The angular frequencies of the motion are: $\omega_{xo}\ \omega_{yo}$ and ω_{so}, the tunes are $\nu_{x0} = \frac{\omega_{x0}}{\omega_0}$, etc. where ω_0 is the revolution frequency.

The reference particle's motion (sometimes called the synchronous particle) is determined by the co-ordinate $\vec{s}$. All other particle motion can be described relative to the reference particle. Six co-ordinates are required to completely describe the motion of the particle; $\{x, x', y, y', z, \delta\}$ where, $x' = \frac{dx}{ds}$, $y' = \frac{dy}{ds}$ and $\delta = \frac{\Delta p}{p}$ is the relative momentum error of the particle. The synchronous particle has co-ordinates $\{0,0,0,0,0,0\}$. The equation for a SHO is simply, $\ddot{x} + \omega^2 x = 0$, so we therefore get three equations of motion for a single particle:[1]

$$\ddot{x} + \left(\frac{\nu_{x0}}{R}\right)^2 x = 0$$

$$\ddot{y} + \left(\frac{\nu_{y0}}{R}\right)^2 y = 0$$

$$\ddot{z} + \left(\frac{\nu_{z0}}{R}\right)^2 z = 0$$

The first two equations describe the simple harmonic property of the transverse betatron oscillations of the particles. The third equation is derived by combining two equations in z and δ to get the longitudinal synchrotron oscillation. These equations describe the un-perturbed motion of the beam. Based on this model you can study various stability problems by introducing perturbations. For example adding non-linear terms to the RHS of these equations leads to the study of single particle non-linear dynamics. We are interested in the perturbations caused by the collective effects of the electro-magnetic fields of the particles.

1.4 Fields of a Moving Charge

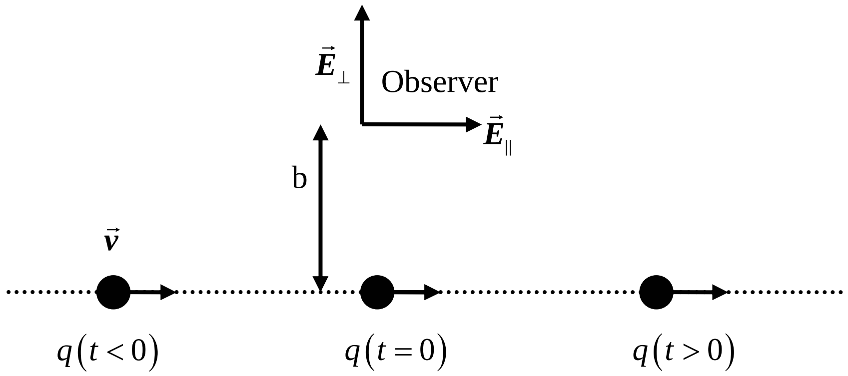

Figure 2: schematic of charge, q, moving past a fixed observer.

For a charge q moving with a velocity $\vec{\boldsymbol{v}}$ (Figure 2) an observer at a distance b normal to $\vec{\boldsymbol{v}}$ will see the following fields, (expressed in terms of the arbitrary basis vectors $\vec{\boldsymbol{e}}_\perp$ and $\vec{\boldsymbol{e}}_\parallel$ that are parallel and perpendicular to $\vec{\boldsymbol{v}}$):[3]

$$\vec{\boldsymbol{E}}_\parallel = -\frac{q}{4\pi\varepsilon_0}\frac{\gamma v t}{\left[b^2+\left(\gamma v t\right)^2\right]^{\frac{3}{2}}}\vec{\boldsymbol{e}}_\parallel$$

$$\vec{\boldsymbol{E}}_\perp = \frac{q}{4\pi\varepsilon_0}\frac{\gamma b}{\left[b^2+\left(\gamma v t\right)^2\right]^{\frac{3}{2}}}\vec{\boldsymbol{e}}_\perp$$

$$\vec{\boldsymbol{B}}_\perp = \frac{\vec{\boldsymbol{v}}\times\vec{\boldsymbol{E}}_\perp}{c^2}$$

The magnetic field has no components in the longitudinal or radial directions. By inspection, the amplitude of $\vec{\boldsymbol{E}}_\perp$ must be an even function of time and a maximum at $t=0$. Similarly $\vec{\boldsymbol{E}}_\parallel$ must be an odd function. As the energy of the particle increases $\vec{\boldsymbol{E}}_\parallel$ decreases, whilst the radial electric $\left(E_r\vec{\boldsymbol{e}}_r = \vec{\boldsymbol{E}}_\perp\right)$ and azimuthal magnetic field $\left(B_\theta\vec{\boldsymbol{e}}_\theta = \vec{\boldsymbol{B}}_\perp\right)$ both increase. If we let $\gamma\to\infty$ (the ultra-relativistic limit) then the fields shrink to a δ function and the only non-vanishing fields that remain are:

$$E_r = \frac{q}{4\pi\varepsilon_0}\delta\left(z-ct\right)$$

$$B_\theta = \frac{E_r}{c}$$

This field distribution is often called a '*pancake*'' distribution, the field components in front and behind the beam vanish. In the limit of $v=c$ the beam is '*frozen*' longitudinally. For a beam in a vacuum chamber the field interacts with the wall of the chamber. This interaction leads to different fields occurring due to discontinuities and tapers of the pipe and due to the finite conductivity of the pipe wall material.

1.5 WAKE FORCES

If we have a cylindrical geometry with a test charge, q_b, following the source charge q_e, the source charge has the position vector $\{\vec{\boldsymbol{r}}_e, z_e\}$ and the test charge $\{\vec{\boldsymbol{r}}_b, z_b\}$.* We can then write down the Lorentz force acting on the test particle:

* In general, where obvious, the basis vectors will be omitted from equations but still implied

$$\vec{\boldsymbol{F}}\left(\vec{\boldsymbol{r}}_b, z_b, t\right) = q\left[\vec{\boldsymbol{E}}\left(\vec{\boldsymbol{r}}_b, z_b, t\right) + \vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}\left(\vec{\boldsymbol{r}}_b, z_b, t\right)\right]$$

The $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields produced by the source are found by solving Maxwell's equations with the required boundary conditions. After passing through the structure the energy and transverse momentum of both charges are modified. In order to solve the necessary equations some assumptions have to be made. We will assume that the change in trajectory of the particle through the section of interest is small. This means that the variation in the fields due to the perturbation of the particles motion is negligible. Making this assumption allows us to simplify the analysis of the forces acting by allowing them to be treated as impulses, (i.e. we can use the time integrated force along the structure to describe the beam dynamics caused by the wake-fields). These forces are known as wake forces. The wake force on the test charge is:

$$\vec{\boldsymbol{F}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, s, t\right) = q\left[\vec{\boldsymbol{E}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z = vt - s, t\right) + v\vec{\boldsymbol{e}} \times \vec{\boldsymbol{B}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z = vt - s, t\right)\right]$$

The change in momentum of the test particle, Δp, is then just equal to the impulse:

$$\Delta p\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, s\right) = \int_L \vec{\boldsymbol{F}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, s, t\right) dt$$

1.6 WAKE FUNCTIONS

As s, z and t are not independent variables the integration w.r.t. time can be replaced with an integration w.r.t. z, the longitudinal position of the source charge, by the relation $s = z - ct$. This change of variables leads to the definition of the "Wake-Function."

$$\vec{\boldsymbol{w}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, s\right) = -\frac{1}{q}\int_{-\infty}^{\infty}\left[\vec{\boldsymbol{E}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z, t\right) + v\vec{\boldsymbol{e}}_{\boldsymbol{z}} \times \vec{\boldsymbol{B}}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z, t\right)\right] dz$$

At a given distance the longitudinal component of the wake function, $w_{||}$, describes the energy gain (or loss) of the test charge. If the source and test charge have the same sign then the negative sign implies that a positive wake function will create a decelerating electric field component.

The energy lost by the source charge is the work done by the longitudinal electromagnetic force along the structure. At $s = 0$ this is related to the wake function by the relation:

$$\Delta E = q_e^2 k_{||} \text{ where } k_{||} = -w_{||}$$

$k_{||}$ is called the loss factor. The loss factor is a property of the environment and not of the beam.

The transverse component of the wake function, $w_\perp$, gives a kick to the trailing particle (i.e. there is a momentum change normal to $\vec{\boldsymbol{e}}_z$).

1.7 WAKE POTENTIALS

For a charge density ρ moving with a constant velocity $\vec{\boldsymbol{v}}$ parallel to the longitudinal axis the wake potential at a distance s from a reference particle in the bunch is obtained by convoluting the charge density with the wake function:

$$\vec{\boldsymbol{w}}^{\rho}\left(\vec{\boldsymbol{r}}_e, s\right) = \int_{-\infty}^{\infty} \rho\left(s'\right) \vec{\boldsymbol{w}}^{\delta}\left(\vec{\boldsymbol{r}}_e, s - s'\right) ds'$$

and ρ is normalised such that:

$$\int_{-\infty}^{\infty} \rho\left(z\right) dz = 1$$

From this definition it is clear that the wake function for the δ-function source $\vec{\boldsymbol{w}}^{\delta}$ is just the Green's function for the wake potential of the charge distribution $\vec{\boldsymbol{w}}^{\rho}$.

1.8 THE FUNDAMENTAL THEORY OF BEAM LOADING

The energy loss of the bunch is obtained by integrating the wake potential weighted by the charge distribution. If we consider a charge distribution of length σ_z then the energy loss is:

$$k_{\parallel}\left(\sigma_z\right) = -\int_{-\infty}^{\infty} \rho\left(s, \sigma_z\right) \vec{\boldsymbol{w}}_{\parallel}^{\rho}\left(s\right) ds = -\int_{-\infty}^{\infty} \rho\left(s, \sigma_z\right) ds \int_{-\infty}^{\infty} \rho\left(s', \sigma_z\right) \vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(\vec{\boldsymbol{r}}_e, s - s'\right) ds'$$

If the bunches are sufficiently short that the wake function, $\vec{\boldsymbol{w}}_{\parallel}^{\delta}$ can be approximated to $\vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(0^+\right)$ we get:[1]

$$\begin{aligned}
k_{\parallel}\left(\sigma_z\right) &\approx -\vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(0^+\right) \int_{-\infty}^{\infty} \rho\left(s, \sigma_z\right) ds \int_{-\infty}^{s'} \rho\left(s', \sigma_z\right) ds' \\
&= -\vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(0^+\right) \int_{-\infty}^{\infty} \rho\left(s\right) ds \int_{-\infty}^{s'} \rho\left(s'\right) ds' \\
&= -\vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(0^+\right) \int_{-\infty}^{\infty} ds \frac{1}{2} \frac{d}{ds} \left(\int_{-\infty}^{s'} \rho\left(s'\right) ds' \right)^2 \\
&= -\frac{\vec{\boldsymbol{w}}_{\parallel}^{\delta}\left(0^+\right)}{2} \left| \left(\int_{-\infty}^{s'} \rho\left(s'\right) ds' \right)^2 \right|_{-\infty}^{\infty}
\end{aligned}$$

$$= -\frac{\vec{w}_{\parallel}^{\delta}\left(0^{+}\right)}{2}$$

Consequently the loss factor for short bunches is just half the wake potential at $s = 0^{+}$. When multiplied by the total charge in the bunch this gives the total amount of energy lost by a bunch to the environment. This relationship is known as the fundamental theory of beam loading. The factor of a half can be understood qualitatively because a charge in a bunch will only see a wake produced by the charges in front of it and for a Gaussian bunch the average charge will only see half the charges in the bunch in front of it.

1.9 Sections of Finite Length

In general the integral over the infinite path length has to be replaced by a finite integral:

$$\vec{w}\left(\vec{r}_b, \vec{r}_e, s\right) = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\vec{E}\left(\vec{r}_b, \vec{r}_e, z, t\right) + v\vec{e}_z \times \vec{B}\left(\vec{r}_b, \vec{r}_e, z, t\right)\right]dz$$

Wake functions that are created by geometrical distortions or discontinuities of a real vacuum vessel† are generally very complicated functions of z and t. The integrals can be made much simpler if we first consider simple geometries. For an infinitely long uniform pipe the fields propagate with the bunch as a function of s. If we assume that the wake forces are constant along the vessel and the wake-potential is linear with L then we can write:

$$\vec{w}\left(\vec{r}_b, \vec{r}_e, s\right) = -\frac{L}{qq_e}\vec{F}\left(\vec{r}_b, \vec{r}_e, s\right)$$

This can be differentiated to give the wake function per unit length:

$$\frac{d\vec{w}\left(\vec{r}_b, \vec{r}_e, s\right)}{dL} = -\frac{1}{qq_e}\vec{F}\left(\vec{r}_b, \vec{r}_e, s\right)$$

When the beam passes through structures before the section of interest other wakefields will be created. Therefore in reality there will be electromagnetic fields that enter the section under consideration. In constructing the wake function per unit length these electromagnetic fields have been neglected.

1.10 Panofsky-Wenzel Theorem

Given the longitudinal electric field of the multipole‡ beam the transverse wakefields can be calculated by a simple integration. For example, for the dipole

† In a real accelerator there are RF cavities, beam monitors, bellows etc.
‡ See section 1.13 for an explanation of multipole expansions of a beam.

mode of a beam displaced by an amount δr in the r direction the transverse force on a test particle (per test particle charge) is:[4]

$$F_r^{dip} = -\frac{1}{r}\int_0^s E_z^{dip} ds$$

This is a consequence of the Panofsky-Wenzel theorem.[5] This theorem states that: the transverse gradient of the longitudinal wake potential is equal to the longitudinal gradient of the transverse wake potential. Symbolically:

$$\nabla_\perp \int_{-L/2}^{L/2} dsF_\parallel = \frac{\partial}{\partial z}\int_{-L/2}^{L/2} dsF_\perp$$

This theorem is of great importance and so a derivation is worthwhile. The theorem can be proved from Faraday's law (and consequently is valid for relativistic and non-relativistic particles). First Faraday's law,

$$\frac{\partial}{\partial t}\vec{\boldsymbol{B}} + \vec{\nabla}\times\vec{\boldsymbol{E}} = 0$$

needs to be de-composed into longitudinal and transverse components. For example the electric field can be expressed as:

$$\vec{\boldsymbol{E}} = \vec{\boldsymbol{E}}_\parallel + \vec{\boldsymbol{E}}_\perp$$

where:

$$\vec{\boldsymbol{E}}_\parallel = \vec{\boldsymbol{e}}_{\boldsymbol{z}} E_z$$
$$\vec{\boldsymbol{E}}_\perp = \left(\vec{\boldsymbol{e}}_{\boldsymbol{z}} \times \vec{\boldsymbol{E}}\right)\times \vec{\boldsymbol{e}}_{\boldsymbol{z}}$$

and $\vec{\boldsymbol{e}}_{\boldsymbol{z}}$ is a unit vector in the z direction, a similar definition is true for the magnetic field $\vec{\boldsymbol{B}}$. The set of four Maxwell equations now become six in terms of the transverse and parallel components. The relevant part of Faraday's law is:[3]

$$\vec{\boldsymbol{e}}_z \times \frac{\partial}{\partial t}\vec{\boldsymbol{B}}_\perp = \frac{\partial}{\partial z}\vec{\boldsymbol{E}}_\perp - \vec{\nabla}_\perp E_z$$

The wake function is:

$$\vec{\boldsymbol{w}} = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\vec{\boldsymbol{E}} + v\vec{\boldsymbol{e}}_{\boldsymbol{z}}\times\vec{\boldsymbol{B}}\right]dz$$

the partial derivative w.r.t. to s is taken and only the transverse components are considered:

$$\frac{\partial}{\partial s}W_{\perp} = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\frac{\partial}{\partial s}\vec{\boldsymbol{E}}_{\perp} + v\vec{\boldsymbol{e}}_{\boldsymbol{z}} \times \frac{\partial}{\partial s}\vec{\boldsymbol{B}}_{\perp}\right]dz$$

Noting that $\frac{1}{v}\frac{\partial}{\partial t} = \frac{\partial}{\partial s}$ and re-writing the velocity as a derivative and cancelling terms gives:

$$\frac{\partial}{\partial s}W_{\perp} = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\frac{1}{v}\frac{\partial}{\partial t}\vec{\boldsymbol{E}}_{\perp} + \vec{\boldsymbol{e}}_{\boldsymbol{z}} \times \frac{\partial}{\partial t}\vec{\boldsymbol{B}}_{\perp}\right]dz$$

Faraday's law can now be used to substitute for $\vec{\boldsymbol{B}}_{\perp}$

$$\frac{\partial}{\partial s}W_{\perp} = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\left(\frac{1}{v}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right)\vec{\boldsymbol{E}}_{\perp} - \vec{\nabla}_{\perp}E_z\right]dz$$

the partial derivative operators can be substituted for an exact derivative by recalling the definition of a partial derivative for a function $F(z,t)$:

$$dF(z,t) = \frac{\partial}{\partial t}Fdt + \frac{\partial}{\partial z}Fdz$$

where $F(z,t)$ is an arbitrary function and so can be removed to give, with a little re-arrangement:

$$\frac{d}{dz} = \frac{1}{v}\frac{\partial}{\partial t} + \frac{\partial}{\partial z}$$

our expression is now:

$$\frac{\partial}{\partial s}W_{\perp} = -\frac{1}{q}\int_{-L/2}^{L/2}\left[\frac{d}{dz}\vec{\boldsymbol{E}}_{\perp} - \vec{\nabla}_{\perp}E_z\right]dz$$

The integral of $\frac{\vec{\nabla}_{\perp}E_z}{q}$ is just the longitudinal wake function giving:

$$\frac{\partial}{\partial s}W_{\perp} = \vec{\nabla}_{\perp}W_{\parallel} - \left[\frac{\vec{\boldsymbol{E}}_{\perp}}{q}\right]_{-L/2}^{L/2}$$

If the transverse electric fields are equal at the beginning and end of the structure considered then

$$\frac{\partial}{\partial s} W_{\perp} = \vec{\nabla}_{\perp} W_{\parallel}$$

This is the Panofsky-Wenzel theorem.

1.11 IMPEDANCE

The wake potential is a function of distance, s. In the ultra-relativistic case the particles move with a constant velocity, $v = c$, this means that we can formulate our equations as functions of time or space. To do this we need only introduce a change of variables and take a Fourier transform. The time difference, τ, between the source and test charge is simply, $\tau = s/v$, and the Fourier transform of the longitudinal wake function w.r.t τ defines the impedance (sometimes called the coupling impedance):

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} d\tau w_{\parallel}(\tau) e^{-i\omega\tau}$$

and the inverse Fourier transforms.

$$w(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega Z_{\parallel}(\omega) e^{i\omega\tau}$$

The wake function in the time domain or the impedance in the frequency domain describes the interaction of the beam with the environment.

1.12 PROPERTIES OF IMPEDANCES.

It can be useful to describe some of the properties of impedances.

As the wake potential is a function of s it necessarily is a real function. The impedance, however, is a complex function, and can be decomposed into it's real and imaginary parts:

$$Z_{\parallel}(\omega) = Z_{\parallel,\mathrm{Re}}(\omega) + iZ_{\parallel,\mathrm{Im}}(\omega)$$

The real part of the impedance is always an even function of the frequency and the imaginary part is an odd function. This can be understood if we expand out the exponential in the definition of the impedance:

$$w(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \left[Z_{\parallel,\mathrm{Re}}(\omega) + iZ_{\parallel,\mathrm{Im}}(\omega) \right] e^{i\omega\tau}$$

$$w(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \left[\left(Z_{\parallel,\mathrm{Im}} Sin(\omega\tau) - Z_{\parallel,\mathrm{Re}} Cos(\omega\tau) \right) - i \left(Z_{\parallel,\mathrm{Im}} Cos(\omega\tau) + Z_{\parallel,\mathrm{Re}} Sin(\omega\tau) \right) \right]$$

The imaginary parts go to zero for all values of s if:

$$Z_{\|,\mathrm{Re}}(\omega) = Z_{\|,\mathrm{Re}}(-\omega)$$
$$Z_{\|,\mathrm{Im}}(\omega) = -Z_{\|,\mathrm{Im}}(-\omega)$$

Even an odd wake functions can then be introduced which must correspond to the real and imaginary parts of the impedance:

$$w(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\|,\mathrm{Re}}(\omega) Cos(\omega\tau)\, d\omega$$
$$w(\tau) = -\frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\|,\mathrm{Im}}(\omega) Sin(\omega\tau)\, d\omega$$

For a purely imaginary impedance the integral over s is zero. This means that the force on the bunch must then be of a purely conservative nature. (An example of this is the impedance of the longitudinal space-charge effect in a perfectly conducting vessel.) This means that only the even part of the impedance contributes to the energy loss of a particle. [§]

$$k_{\|} = -w_{\|} = \frac{1}{\pi}\int_{-\infty}^{\infty} Z_{\|,\mathrm{Re}}(\omega)\, d\omega$$

The real part of the impedance describes the frequency spectrum of energy loss, it also known as resistance. The imaginary part of the impedance is also known as reactance.

For particles where $v = c$ the wake function must vanish for negative distances, i.e. when our test particle is in front of the source charge, to safeguard causality. This can only be true if the even and odd parts cancel. In terms of impedance we can write this as:

$$\int_{-\infty}^{\infty} Z_{\|,\mathrm{Re}}(\omega) Cos(\omega\tau)\, d\omega = \int_{-\infty}^{\infty} Z_{\|,\mathrm{Im}}(\omega) Sin(\omega\tau)\, d\omega$$

This relation is equivalent to performing a Hilbert transform between the real and imaginary parts of the impedance.[**] This is the same as the relationships between the real and imaginary parts of a network impedance. This means that a network can be found that exactly replicates the impedance of a vacuum vessel. For a single mode

---

[§] The factor of two has been cancelled due to the fundamental theorem of beam loading.

[**] A Hilbert transform, $\mathbf{H}$, is an integral transform in which all the phases of a signal are transformed by $-\frac{\pi}{2}$ radians. E.g. for $f(x) = Sin(x)$ the Hilbert transform $\mathbf{H}\left[f(x)\right] = Cos(y)$ and for $f(x) = Cos(x)$ then $\mathbf{H}\left[f(x)\right] = -Sin(y)$ etc.

a parallel LCR resonance circuit suffices. The excitation of the cavity is analogous to the excitation of the current source of the circuit.

1.13 MULTIPOLE MODE EXPANSION OF A BEAM

Consider an amount of charge, q_e, moving down an axially symmetric pipe, a cylindrical co-ordinate system can be used (r,θ,z) with the symmetry axis of the structure at $r=0$. The charge is at a radius r_e in the $\theta=0$ direction. The (pancake) charge distribution, ρ_e, can be expressed as:

$$\rho_e(r,\theta,z)=\frac{q_e}{r}\delta(r-r_e)\delta(\theta)\delta(z-z_e)$$

As the charge must be periodic in the θ direction we can write:

$$\delta(\theta)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}e^{im\theta}$$

The exponential can be written in terms of cosine functions if the range of the summation is changed to 0 to ∞ giving:

$$\delta(\theta)=\frac{1}{2\pi}\sum_{m=0}^{\infty}\left(2-\delta_{m,0}\right)Cos(m\theta)$$

Where $\delta_{m,0}$ is the Kronecker δ. The multipole expansion of the charge density is then:

$$\rho_e=\sum_{m=0}^{\infty}\rho_m$$

where the ρ_m distribution is:

$$\rho_m=q_e\frac{\left(2-\delta_{m,0}\right)}{2\pi r}\delta(r-r_e)\delta(z-z_e)Cos(m\theta)$$

The charge moving down the pipe can be thought of as a superposition of infinitely thin rings with a $Cos(m\theta)$ angular dependence. The electromagnetic field carried by the $Cos(m\theta)$ ring is obtained by solving Maxwell's equations with the proper boundary conditions. The monopole term, where $m=0$, describes a charged ring with uniform density, the dipole term consists of a positively charged half ring and a negatively charged half ring. Figure 3 shows the electric field in the pancake region for the first three terms of the multipole expansion.[6] The electric field is always normal to the vessel boundary, but not to the ring beam. The field is not continuous across the ring beam boundary.

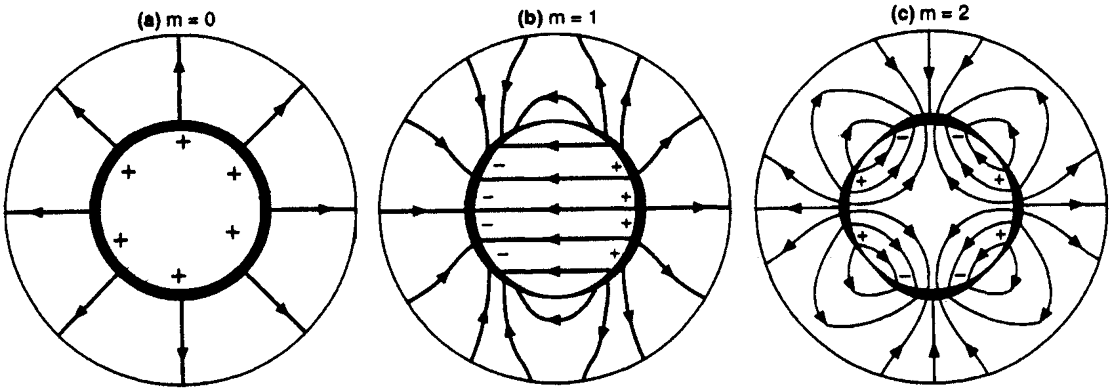

Figure 3: electric field pattern and ring charge distribution associated with a $Cos(m\theta)$ ring beam, monopole, dipole and quadrupole patterns are shown.

1.14 Calculating Wake-Fields

A typical method for calculating wakefields is to solve Maxwell's equations as a system of coupled partial differential equations of charged particles with $v = c$. The charge distribution is written as an expansion of multipole terms to eliminate the azimuthal dependence of the charge distribution. The systems of equations for each multipole are decoupled and so can be linearly combined due to the principle of super-position. The translational invariance of a longitudinally uniform pipe can be utilised by considering the variable $s = vt - z$. (Depending upon which direction the charge is imagined to go (e.g. left to right or right to left?) this variable can equally be defined as $s = vt + z$.) By transforming the equations into the frequency domain the system of partial differential equations becomes a system of ordinary differential equations. By taking boundary conditions at the vessel interface a set of matched and uniquely defined solutions can be arrived at.

2. Resistive Wall Wakefields of a Round Pipe

We have seen that in general the fields of the beam exert forces on the particles. The general formalism which describes these effects on the trajectories of the particles is given by the concepts of wake functions and impedances. We will now concentrate the discussion on an axially symmetric "round-pipe" system. Our particles will now go from right to left. (Both conventions are used and it is useful to be familiar with either beam direction.)

2.1 Wake Functions

The wake functions are obtained by integration of the electromagnetic forces acting on a test charge at a distance s behind the exciting one. The test charge has a transverse offset $\vec{r}_e$ from the axis of the cylinder and the exciting charge has a transverse offset given by $\vec{r}_b$ (Figure 4).

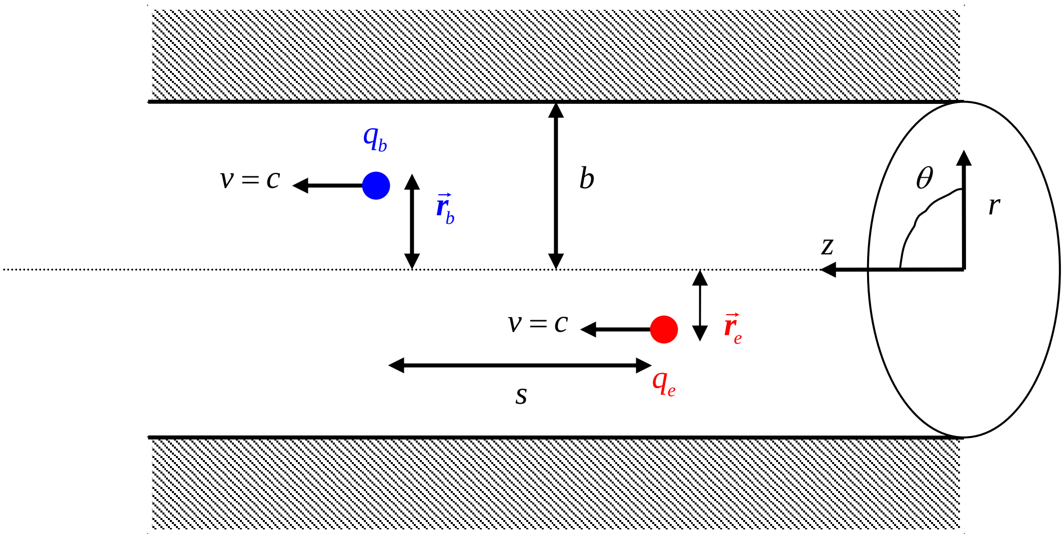

Figure 4: schematic of axially-symmetric geometry and particle motion considered.

Since the magnetic field is perpendicular to the direction of the particle motion it does not contribute and the longitudinal wake function is obtained by integrating over the electric field component E_z, normalised by the charge q.

$$w_{\parallel}\left(\vec{r}_b,\vec{r}_e,s\right)=-\frac{1}{q}\int_{-\infty}^{\infty}E_z\left(\vec{r}_b,\vec{r}_e,z,t=\frac{z+s}{c}\right)dz$$

The minus sign means that the wake function is positive for a decelerating electric field component. The dimensions of the wake function are $\frac{V}{C}=\frac{\Omega}{s}$, in accelerator applications the units $\frac{V}{pC}=10^{12}\frac{V}{C}$ are often used. The longitudinal wake function is defined as the change of energy ΔE of a test charge of magnitude e a distance s behind the point charge q. Simply, $\Delta E=-eqw_{\parallel}\left(s\right)$.

In an infinitely long homogenous structure the field dependence from the longitudinal co-ordinate and from the time is $s=z-ct$. This means that the co-ordinates for the relevant component of the electric field are, $E_z\left(\underset{\sim}{r}_b,\underset{\sim}{r}_e,s\right)$, there is no dependence on z. Therefore we can define the longitudinal wake-function per unit length, $\tilde{w}_{\parallel}$, as:

$$\tilde{w}_{\parallel}\left(\vec{r}_b,\vec{r}_e,s\right)=\lim_{L\to\infty}\left\{-\frac{1}{qL}\int_{-L/2}^{L/2}E_z\left(\vec{r}_b,\vec{r}_e,s\right)dz\right\}$$

This expression can easily be integrated to give:

$$\tilde{w}_{\parallel}\left(\vec{r}_b,\vec{r}_e,s\right)=-\frac{1}{q}E_z\left(\vec{r}_b,\vec{r}_e,s\right)$$

2.2 LONGITUDINAL IMPEDANCE

The concept of impedance is introduced to relate the beam current to the total induced voltage along the beam trajectory. The voltage is simply the (normalised) integral of the electro-magnetic force acting on a test charge. For a beam moving at a constant velocity, at an offset $\underset{\sim}{r}_b$, from the z axis only the force of the electric field is relevant in the longitudinal direction, (as the magnetic fields are perpendicular to the motion). The longitudinal impedance is:

$$Z_{\|}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, \omega\right) = -\frac{1}{I}\int_{-\infty}^{\infty} E_z\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z, \omega\right) e^{ikz} dz$$

where the current and field components are to be evaluated at the angular frequency ω. In a purely homogenous structure $E_z\left(\vec{\boldsymbol{r}}_b, \omega, z\right) = E_z\left(\vec{\boldsymbol{r}}_b, \omega\right) e^{-kz}$, and the impedance per unit length can be introduced, $\tilde{Z}_{\|}$:

$$\tilde{Z}_{\|}\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, \omega\right) = \lim_{L\to\infty}\left\{-\frac{1}{IL}\int_{-L/2}^{L/2} E_z\left(\vec{\boldsymbol{r}}_b, \vec{\boldsymbol{r}}_e, z, \omega\right) e^{ikz} dz\right\} = -\frac{1}{I} E_z\left(\vec{\boldsymbol{r}}_b, \omega, z\right)$$

The dimensions of impedance are Ω and impedance per unit length $\frac{\Omega}{m}$.

If we look at the definitions of the impedance and the wake functions it is clear that for $v = c$ the longitudinal impedance is the Fourier transform of the wake function, giving two Fourier transform pairs:

$$Z_{\|}(\omega) = \int_{-\infty}^{\infty} w_{\|}(\tau) e^{-i\omega\tau} d\tau \quad w_{\|}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\|}(\omega) e^{i\omega\tau} d\omega$$

$$\tilde{Z}_{\|}(\omega) = \int_{-\infty}^{\infty} \tilde{w}_{\|}(\tau) e^{-i\omega\tau} d\tau \quad \tilde{w}_{\|}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{Z}_{\|}(\omega) e^{i\omega\tau} d\omega$$

2.3 LONGITUDINAL IMPEDANCE/WAKE

The longitudinal impedance of a resistive wall expressed as an expansion of the multipole modes of the beam is:

$$\tilde{Z}_{\|}\left(k, r_b, r_e, \theta\right) = \frac{Z_0}{\pi b}\sum_{m=0}^{\infty}\frac{\cos(m\theta)}{1+\delta_{0,m}}\left(\frac{r_e r_b}{b^2}\right)^m\left[-\left(1+\delta_{0,n}\right)\frac{i\lambda}{k} + \frac{ikb}{n+1}\right]^{-1}$$

Where:

- b is the pipe radius
- $m = 0,1,2...$

- $Z_0 = 120\pi\ \Omega$ is the impedance of free space
- $\lambda^2 = i\sigma_c k Z_0$
- $\delta_{0,m}$ is the Kronecker δ function

For an on axis electron bunch only the monopole, $m = 0$, mode is non zero, (meaning we are only considering $\tilde{Z}_{\parallel}^{0}$). The monopole impedance can be written in terms of the characteristic distance of the pipe, s_0 and the dimensionless variable, κ, where:

$$s_0 = \sqrt[3]{\frac{2cb^2\varepsilon_0}{\sigma_c}}$$

$$\kappa = ks_0$$

to give:

$$\tilde{Z}_{\parallel}^{0}(\kappa) = \frac{Z_0 s_0}{2\pi b^2}\left[\frac{1 - i\,sign(\kappa)}{|\kappa|^{1/2}} + \frac{i\kappa}{2}\right]^{-1}$$

Where:

$$sign(x) = \begin{cases} -1 & for \quad x < 0 \\ 0 & for \quad x = 0 \\ 1 & for \quad x > 0 \end{cases}$$

The longitudinal wake function is then derived by substituting $\tilde{Z}_{\parallel}^{0}(\kappa)$ into $\tilde{w}_{\parallel}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{Z}_{\parallel}(\omega)e^{i\omega\tau}d\omega$ and performing the integration in the complex plane. The result is:[7]

$$\tilde{w}_{\parallel}(s) = -\frac{4}{\pi\varepsilon_0 b^2}\left\{\frac{1}{3}e^{-\frac{s}{s_0}}\cos\left(\sqrt{3}\frac{s}{s_0}\right) - \frac{\sqrt{2}}{\pi}\int_0^{\infty}\frac{x^2}{x^6+8}e^{-x^2\frac{s}{s_0}}dx\right\} \text{ for } s > 0$$

$$\tilde{w}_{\parallel}(s) = \frac{\tilde{w}_{\parallel}(0_-)}{2} = -\frac{1}{2\pi\varepsilon_0 b^2} \text{ for } s = 0 \qquad (1)$$

$$\tilde{w}_{\parallel}(s) = 0 \text{ for } s < 0$$

If we are dealing with long range wakes then $s >> s_0$ and the equation can be simplified to:

$$\tilde{w}_{\parallel}(s) = -\frac{1}{4\pi\varepsilon_0\sqrt{2\pi}b^2}\left(\frac{s_0}{s}\right)^{3/2} \tag{2}$$

The wake potential for a bunch is obtained by the convolution of the wake function with the longitudinal charge distribution. A Gaussian bunch with an rms length of σ gives:

$$\tilde{w}_{\parallel}^{gauss}(s) = \frac{1}{\sqrt{2\pi}\sigma}\int_0^{\infty}\tilde{w}_{\parallel}^{\delta}(s')e^{-\frac{(s-s')^2}{2\sigma^2}}ds' \tag{3}$$

The loss factor is given by:

$$k_{\parallel} = -\int_{-\infty}^{\infty}\rho(s)\tilde{W}_{\parallel}(s)ds \tag{4}$$

From which the rms energy spread of the bunch, σ_ε, can be calculated:

$$\sigma_\varepsilon = eN\left|\int_{-\infty}^{\infty}\left(\rho(s)\tilde{W}_{\parallel}^2(s) - k_{\parallel}^2\right)ds\right|^{1/2} \tag{5}$$

where N is the number of particles and e is their charge. For a bunch travelling on axis and $\sigma_z \gg s_0$ the expression becomes:[8]

$$\sigma_\varepsilon \approx \frac{0.46Nq^2c}{2\pi^2 b(\sigma_z)^{\frac{3}{2}}}\sqrt{\frac{Z_0}{\sigma_0}}$$

where σ_0 is the conductivity of the vessel.

2.4 TRANSVERSE IMPEDANCE/WAKE

The transverse impedance is due to the transverse electromagnetic fields and is:

$$Z_{\perp}(\vec{r},\omega) = \frac{i}{qr_b}\int_{-\infty}^{\infty}\left[\vec{\boldsymbol{E}}_{\perp}(\vec{\boldsymbol{r}},z,\omega) + \vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}_{\perp}(\vec{\boldsymbol{r}},z,\omega)\right]e^{\frac{i\omega z}{v}}dz$$

From this definition of the impedance of the dipole order wake function per unit length in the round resistive pipe has been computed.[9] It is found to be:

$$\tilde{w}_{\perp}(s) = \frac{2s_0 r}{3\pi\varepsilon_0 b^4}\left\{e^{-\frac{s}{s_0}}\left[\sqrt{3}\sin\left(\frac{s\sqrt{3}}{s_0}\right) - \cos\left(\frac{s\sqrt{3}}{s_0}\right)\right] + \frac{12\sqrt{2}}{\pi}\int_0^{\infty}\frac{e^{-x^2\frac{s}{s_0}}}{x^6+8}dx\right\} \tag{6}$$

The wake potential is then found via a convolution with the Gaussian bunch as before. A change of variables is required to evaluate the integral, where $\frac{s'}{\sigma} \to s'$ and $\frac{s}{\sigma} \to s$. This gives the integral normalised to σ:

$$\tilde{w}_{\perp}^{gauss}(s) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^{\infty} \tilde{w}_{\perp}^{\delta}(s') e^{-\frac{(s-s')^2}{2\sigma^2}} ds' = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \tilde{w}_{\perp}^{\delta}(s') e^{-\frac{(s-s')^2}{2}} ds' \qquad (7)$$

A transverse loss factor (sometimes called the kick factor) can then be defined as:

$$k_{\perp} = \frac{1}{r}\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{w}_{\perp}(s') e^{-\frac{s'^2}{2}} ds' \qquad (8)$$

3. SURFACE ROUGHNESS WAKEFIELDS

So far the work has assumed that the pipe is perfectly smooth. Of course this is just an approximation, in reality the surface is not smooth and the degree of roughness will also have an effect on the wakefields generated. One of the most comprehensive models that simulates the effect of the surface roughness of a pipe is the Novokhatski – Timm - Weiland (NTW) model.[10] Here the roughness is modelled as periodic rectangular corrugations of the vacuum chamber walls. The corresponding point longitudinal and transverse wake potentials are then given as:

$$w_z(s) = \frac{Z_0 c}{\pi b^2} Cos(ks)$$

$$w_r(s) = \frac{2Z_0 c}{\pi b^2} Sin(ks)$$

where $k = \sqrt{\frac{2p}{b\delta g}}$, δ is a the rms roughness size, g is a gap and p is a period of corrugations, (see Figure 5). The packing factor of the corrugations, $\alpha = p/g$, can also be defined. The material is assumed to be a perfect conductor.

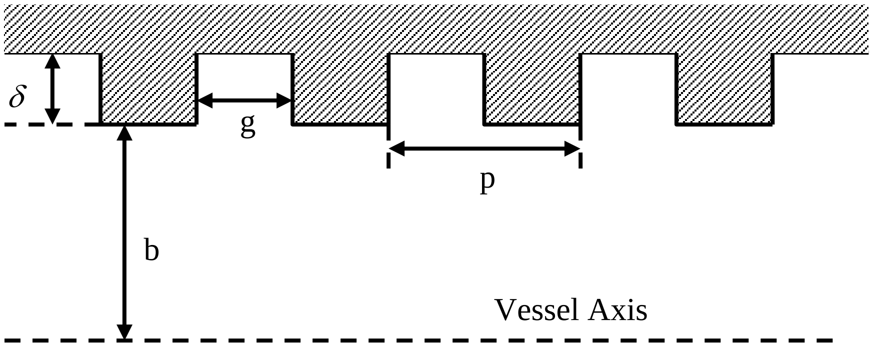

Figure 5: geometry of surface roughness model.

The analytical form of the longitudinal wake function for a Gaussian bunch is then given by:[11]

$$W_z(s) = -\frac{Z_0 c}{4\pi b^2} e^{-s^2/2\sigma_z^2} \left\{ \xi\left(\frac{-is - k\sigma_z^2}{\sigma_z\sqrt{2}}\right) + \xi\left(\frac{-is + k\sigma_z^2}{\sigma_z\sqrt{2}}\right) \right\} \tag{9}$$

where $\xi(x)$ is the complex error function defined as:[12]

$$\xi(x) = e^{-z^2}\left(1 - erf\left(ix\right)\right)$$

and $erf\left(x\right)$ is the error function, defined as:[10]

$$erf\left(x\right) \equiv \frac{2}{\pi}\int_0^x e^{-t^2} dt$$

The long range (asymptotic) form of the longitudinal wake function can be derived from the asymptotic form of the error function. For large arguments:

$$z\sqrt{\pi}e^{z^2}\left(1 - erf\left(z\right)\right) \approx 1$$

In the long range approach $k\sigma_z >> 1$ and the formula is:

$$W_z(s) = \frac{Z_0 c}{3^{3/4}\pi^{3/2}k^2 b^2 \sigma_z^3\sqrt{2}} s e^{-s^2/2\sigma_z^2} \tag{10}$$

and the impedance is inductive (no power losses). However the roughness induces the energy spread within the bunch given by:

$$\sigma_\varepsilon = \frac{Z_0 c}{3^{3/4}\pi^{3/2}\sigma_z^2 k^2 b^2\sqrt{2}} \tag{11}$$

4. EXAMPLE CALCULATIONS FOR THE TESLA UNDULATOR

The wake-field effects for the permanent magnet option for the TESLA undulator for the production of positrons need to be calculated. For the super-conducting device the effects are different due to the low temperatures involved, and the so-called ‘anomalous’ skin effect.[13]

For this calculation pessimistic values will be assumed to give the worst effects. The device will be built in sections and between each section will be a short gap. The gap could be used for diagnostics and magnets. The geometry is not yet decided and each gap could have a tapered vacuum vessel. Such tapering would induce geometric wakefields,[14] however the present work does not account for such effects (although they are thought to be small[15]) and so consequently a purely cylindrical vacuum vessel will be considered with the following parameters:

Parameter	Unit	Value
Internal Diameter	**mm**	**3.7**
External Diameter	**mm**	**4**
Copper Conductivity (static)	Ω^{-1} **m**$^{-1}$	**5.9 10**^{7}
Length[††]	**m**	**140**

Table 1: vacuum vessel parameters.

4.1 LONGITUDINAL WAKE-FUNCTION, LOSS FACTOR & ENERGY SPREAD

The longitudinal wake function is calculated using (1), (2) and (3), the σ of the bunch length used is 25 μm. Figure 6 shows the longitudinal wakefield and the long range approximation, the Gaussian bunch profile is also shown. The loss factor $k_{\|}$, (4), was found to be 76.6 eV pC^{-1} m^{-1}. The resistive energy spread, (5), was found to be 79.8 eV pC^{-1} m^{-1}.

[††] The exact length of the device depends upon the length of the undulator sections and the gap see ASTeC-ID-031 “*Calculation of the Power Deposited due to Synchrotron Radiation Incident on the Vessel Wall for the TESLA Positron Undulator*” for examples. Only the effects of the undulator beam pipe will be considered, not the gaps between undulator sections.

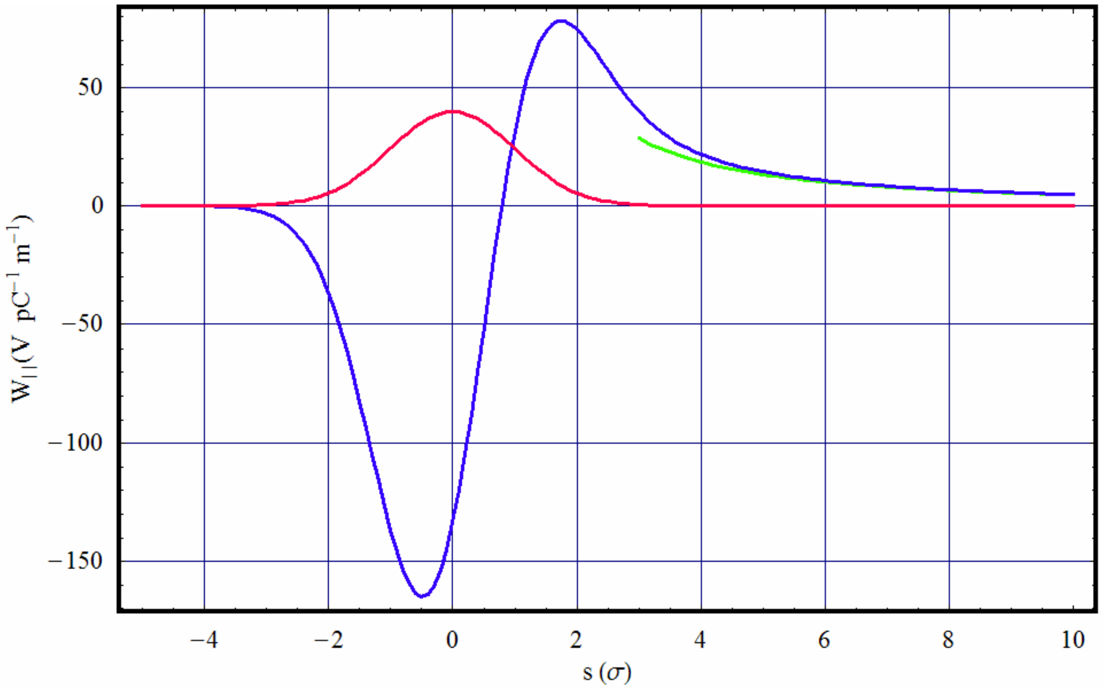

Figure 6: analytic (blue) and long range (green) longitudinal wake potential and the Gaussian bunch (red).

4.2 TRANSVERSE WAKE FUNCTION & LOSS FACTOR

The transverse wake function is calculated using (6) and (7) and is shown in Figure 7.

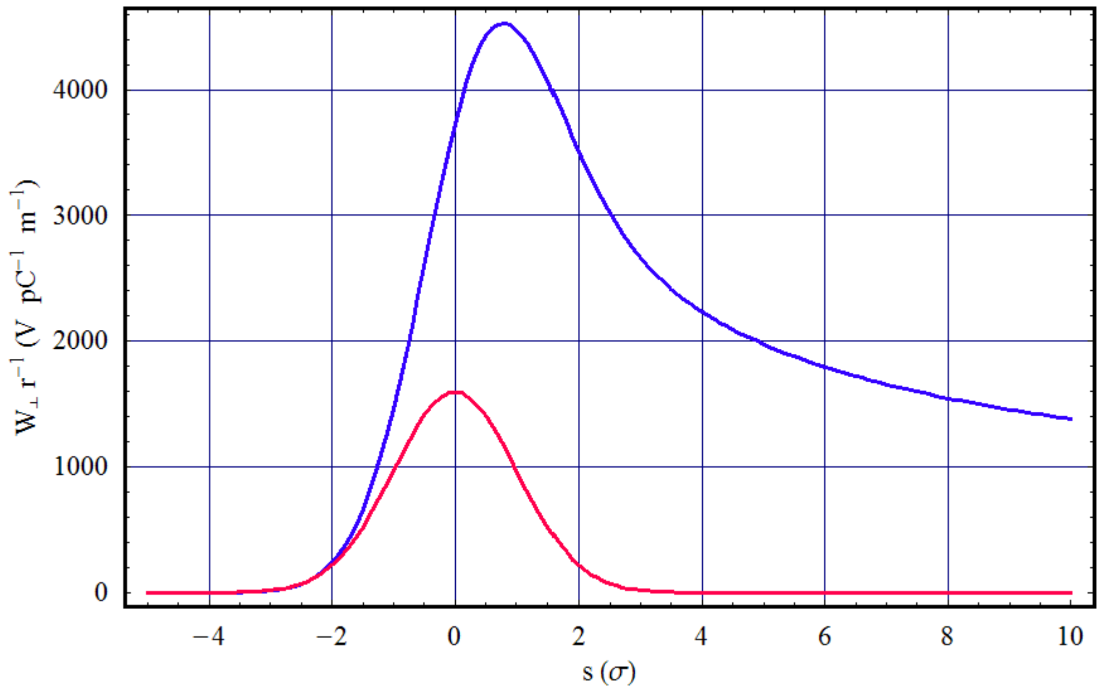

Figure 7: transverse wake potential (blue) for the Gaussian bunch (red).

The transverse loss factor, calculated using (8), is 3.1 keV pC^{-1} m^{-1}.

4.3 SURFACE ROUGHNESS

At the present time we do not know what the surface roughness of a NEG coated vessel will be. Therefore we will assume that the roughness induced energy spread should not exceed that of the longitudinal resistive wake. The value for the resistive energy spread was found to be 79.8 eV pC^{-1} m^{-1}. This means that, using (11), the associated k parameter is ~1.9 10^5. This gives $k\sigma \simeq 4.8$ indicating that the long range approximation is probably valid. For a packing factor of $\frac{p}{g} = 2$ then the allowed roughness is only $d = 58nm$. Figure 8 shows the comparison of the long range formula and the analytic one. There is a small discrepancy but not a significant one.

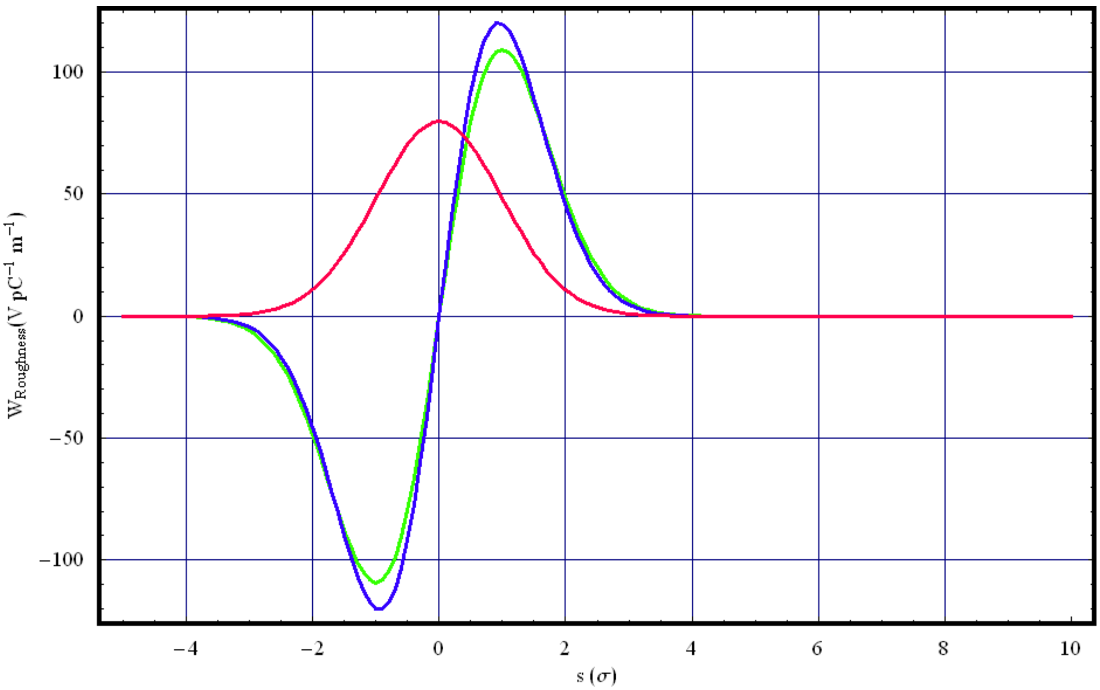

Figure 8: analytic (blue) and long range (green) roughness wake potentials that cause an energy spread of the order ~79.8 eV pC^{-1} m^{-1} and a Gaussian bunch (red).

5. RESULTS

The effects on the TESLA beam can be calculated from the longitudinal and transverse loss factors and the induced energy spread. The material used for the vacuum vessel is also very important. Typically stainless steel or aluminium are used but both have very different conductivities – which has a large affect on the induced wake fields. Table 2 gives the relevant TESLA beam parameters[16] and the conductivities of copper, aluminium, and stainless steel, the three materials that will be considered for the vessel.

Parameter	Unit	Value
Beam Energy, E_0	**GeV**	**250**
# Particles per Bunch, N		**$2\ 10^{10}$**
Bunch Charge	**pC**	**3204**
rms bunch length, σ_z	**μm**	**25**
Copper Conductivity	**$\Omega^{-1}\ m^{-1}$**	**$5.88\ 10^7$**
Aluminium Conductivity	**$\Omega^{-1}\ m^{-1}$**	**$3.62\ 10^7$**
Stainless Steel Conductivity	**$\Omega^{-1}\ m^{-1}$**	**$1.7\ 10^6$**

Table 2: parameters for wake-field calculations of the TESLA beam.

From the parameters from Table 1 and Table 2 the characteristic length, s_0, for each type of material can be calculated shown in Table 3.

Vessel Material	Unit	s_0
Copper	**μm**	**6.8**
Aluminium	**μm**	**7.9**
Stainless Steel	**μm**	**22**

Table 3: characteristic length of the wake fields for a round pipe of diameter 3.7mm for different metals.

Table 3 indicates that for copper and aluminium vessels the long range approximations for the wake fields can be used because $s_0 < \sigma_z$. For stainless steel $s_0 \approx \sigma_z$ and so numerical methods must be used to evaluate the wake field effects for this material. The following results are all obtained using numerical methods.

The resistive effects on the beam, the average energy loss ΔE, the energy broadening $\frac{\sigma_E}{E_0}$ and the average transverse loss factor per unit offset $NeLk_\perp$ can now be calculated for each material, Table 4.

Vessel Material	ΔE (MeV)	$\frac{\sigma_E}{E_0}$	$NeLk_\perp$ (GeV m^{-1})
Copper	**34**	**1.4 10^{-4}**	**1.7**
Aluminium	**44**	**1.8 10^{-4}**	**1.9**
Stainless Steel	**253**	**8.8 10^{-4}**	**7.8**

Table 4: effect of the resistive wall wakefields of the undulator vessel on the TESLA beam.

6. CONCLUSIONS AND FURTHER WORK

For the simple cylindrical vessel geometry discussed the difference between using a stainless steel pipe or copper/aluminium is quite different. The total energy lost by the beam for a copper/aluminium pipe is ~0.015%. For stainless steel ~0.1% of the beam energy is lost.

For the induced energy spread copper/aluminium induce an energy spread that is approximately an order of magnitude less than the nominal 0.1% TESLA beam energy spread.[14] Stainless steel increases the energy spread by an amount approaching the nominal value.

For the transverse loss factor results the induced energy loss of the beam is much less than in the longitudinal case. A beam travelling through the undulator 100 μm off axis will only lose ~ 180 keV for aluminium or copper vessels and ~780 keV for a stainless steel vessel. This is much smaller than the energy lost due to the longitudinal impedance.

The effects of the stainless steel pipe are worse than copper and aluminium pipes, which are very similar. However these results will have to be discussed with the TESLA accelerator physicists to see if the effects are significant for the beam. These discussions will also indicate whether a stainless steel pipe can be used.

The calculations presented here are for an infinitely long ‘round pipe’ and are a very good approximation to a pipe 140m long with 4mm aperture. This vessel will not have those dimensions. It is expected that the undulator will be built in sections ~2m long. In between the undulator sections there could be tapers leading to vacuum pumps and diagnostics. These tapers will lead to geometric wakefields, however to accurately study them the dimensions of the tapers need to be known.

The second approximation in this model that can be improved upon is that the NEG layer has not been considered. New work on the wakefield effects of pipes with thin layers has been presented at EPAC 2004[17] and will be examined in future studies. It is unclear at the moment exactly what the conductivity of the NEG coating is, but recent studies at the ESRF give a value of 2.5 $10^{-5}\Omega$.m. This measurement was carried out with a NEG coating sputtered in the usual way but then squeezed between two plates. This means that the there was no ‘surface roughness’ effect in the measurements.[18, 19]

7. REFERENCES

[1] A.Chao "*The Physics of Collective Beam Instabilities in High Energy Accelerators,*" John Wiley & Sons, 1993
[2] Palumbo *et al* "*Wake Fields and Impedance*" CAS-95-06 Volume 1
[3] J.D. Jackson '*Classical Electro-Dynamics'* Wiley, 1975
[4] K.L.F. Bane 'The Short Range Resistive Wall Wakefields," SLAC/AP-87 June 1991
[5] Panofsky & Wenzel, Rev Sci Intruments **27**, 967, (1956)
[6] Picture from '*The Physics of Collective Beam Instabilities in High Energy Accelerators'* A.Chao, John Wiley & Sons, 1993
[7] K.L.F. Bane '*The Short Range Resistive Wall Wakefields*' SLAC/AP-87 June 1991
[8] Henry & Napoly, CERN-CEA, DPhN/STAS/91-R08 (1991)
[9] K. Bane & M. Sands, '*The Short Range Resistive Wall Wakefield*' SLAC-PUB-95-7074, Dec 1995
[10] K. Bane & A Novokhatskii, 'The Resonator Impedance Model of Surface Roughness Applied to the LCLS Parameters' LCLS-TN-99-1, SLAC-AP-117 March 1999.
[11] M. Dohlus, M.I. Ivanian, and V.M. Tsakanov, 'Surface Roughness Study For The Tesla-Fel' DESY-TESLA-FEL-00-26, Feb 2001
[12] Abramowitz & Stegun '*Handbook of Mathematical Functions'*" Dover Publications 1965.
[13] B. Podobedov '*Image Current Heating in Cold Bore Super-conducting IDs'*" Workshop on Super-Conducting Undulators and Wigglers, ESRF 2003
[14] Onoprienko et al '*Measurement of Resitivity Dominated Collimator Wakefield Kicks at the SLAC*' EPAC 2002
[15] Discussion with J. Sheppard
[16] TESLA Technical Design Report, II-14, March 2001
[17] M. Ivanian, V.M. Tsakanov, '*The Small Gap Undulator Impedance Study*' EPAC 2004
[18] E. Plouviez '*Measurement of the Resistivity of NEG Coating at 14 GHz*' ESRF Diagnostics 02-03 April 2002
[19] Discussion with E. Plouviez