# Implementation of depolarization due to beam-beam effects in the beam-beam interaction simulation tool GUINEA-PIG++

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**Abstract**

Depolarization is a new feature in the beam-beam simulation tool GUINEA-PIG++ (GP++). The results of this simulation are studied and compared with the other beam-beam simulation, CAIN, for the different options of the ILC at 500 GeV.

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# 1 Introduction

Precise knowledge of the polarization state of the beams will be required at future linear colliders such as ILC and CLIC [1]. As for many other beam characteristics, the polarization of the beams is modified by beam-beam effects. Such effects are not analytically calculable and beam-beam interaction simulations are thus required.
A treatment of the corresponding spin dynamics has been implemented in GP++ [2] for the beam particles, complementing that already available in the CAIN simulation [3].

# 2 Depolarization effects during beam-beam interaction

The spin motion in an electromagnetic field is influenced by two effects: the classical spin precession, described by the Thomas-Bargmann-Michel-Telegdi (T-BMT) equation and the spin-flip effect arising from beamstrahlung [4].
The dominant depolarization effect at ILC energies comes from the classical precession of the spin under an electromagnetic field, according to the T-BMT equation:

$$\frac{d\vec{S}}{dt} = -\frac{e}{m\gamma}[(\gamma a+1)\vec{B_T} + (a+1)\vec{B_L} - \gamma(a+\frac{1}{\gamma+1})\beta\vec{e_v} \times \frac{\vec{E}}{c}] \times \vec{S}. \qquad (1)$$

The precession angle is typically $\gamma a$ times the deflection angle, where $a = 0.0011596$ is the anomalous magnetic moment of the electron and $\gamma$ the Lorentz factor.
The Sokolov-Ternov effect, also called spin-flip, occurs when electrons emit synchrotron radiation. The spin of the radiating electron then has a probability to change its state, proportional to the energy of the emitted photon.

# 3 Beam spin state after interaction

GP++, as well as CAIN, do not simulate the interaction of the real number of particles, but use macro-particles. Typically, $10^5$ macro-particles replace $2 \times 10^{10}$ real particles. A macro-particle represents thus a statistical ensemble, to which a polarization can be associated as a probability for the spins to be oriented along a given axis. A polarization defined as (0,0,1) is used as initial condition for all the comparisons in this paper, meaning that 100% of the particles have their spin oriented along the $z$ axis.
An implementation of the two depolarization mechanisms described above, precession and spin-flip, has been realized in GP++ for the beam macro-particles. The GP++ results are compared to CAIN simulations, for the four beam parameter sets of the ILC at 500 GeV center-of-mass energy (see Table 1).

## 3.1 Depolarization from precession

The spin state of the beam is first compared after a full beam-beam interaction, taking into account effects from precession only. As shown in Figure 1, good agreement is

|  | Nominal | Low N | Large Y | Low P |
|---|---|---|---|---|
| N [$10^{10}$] | 2 | 1 | 2 | 2 |
| $\gamma\epsilon_x/\gamma\epsilon_y$ [mm.mrad] | 10/0.04 | 10/0.03 | 10/0.08 | 10/0.036 |
| $\beta_x/\beta_y/$ [mm] | 20/0.4 | 11/0.2 | 11/0.6 | 11/0.2 |
| $\sigma_x/\sigma_y$ [nm] | 639/5.7 | 474/3.5 | 474/9.9 | 474/3.8 |
| $\sigma_z$ [$\mu$m] | 300 | 200 | 500 | 200 |

Table 1: Beam parameters for ILC at 500 GeV center-of-mass energy. $N$ is the number of particles per bunch, $\gamma\epsilon_{x,y}$ respectively horizontal and vertical normalized emittances at the IP, $\beta_{x,y}$ respectively the horizontal and vertical beta functions and $\sigma_{x,y,z}$ respectively the horizontal, vertical and longitudinal sizes of the bunches.

obtained between GP++ (red line) and CAIN (blue line) for the distributions of the polarizations of the beam macro-particles in each of the four options. The corresponding mean depolarizations, $\Delta P$, calculated as $1- < P >$, are displayed in Table 2. The agreement between the two simulations is better than 95%.

| $\Delta P$ [%] from precession | Nominal | Low N | Large Y | Low P |
|---|---|---|---|---|
| GP++ | 0.63 | 0.29 | 1.39 | 1.04 |
| CAIN | 0.64 | 0.29 | 1.46 | 1.08 |
| (CAIN - GP++)/CAIN [%] | 1.56 | 0 | 4.79 | 3.70 |

Table 2: Depolarization due to precession after a full beam-beam interaction, for the four ILC beam parameter sets at 500 GeV center-of-mass energy.

## 3.2 Total depolarization

Beamstrahlung emission induces an additional spin-flip probability. Figure 2 shows the polarization spectra after the full beam-beam interaction, now taking into account both of the two depolarization effects described above. Mean values are compared in Table 3 [5]. The Sokolov-Ternov mechanism increases the depolarization by about 20% on average for all ILC options.
Figure 3 shows the depolarization as a function of macro-particle energy. One can clearly see the energy-dependance expected from the Sokolov-Ternov spin-flip probability.

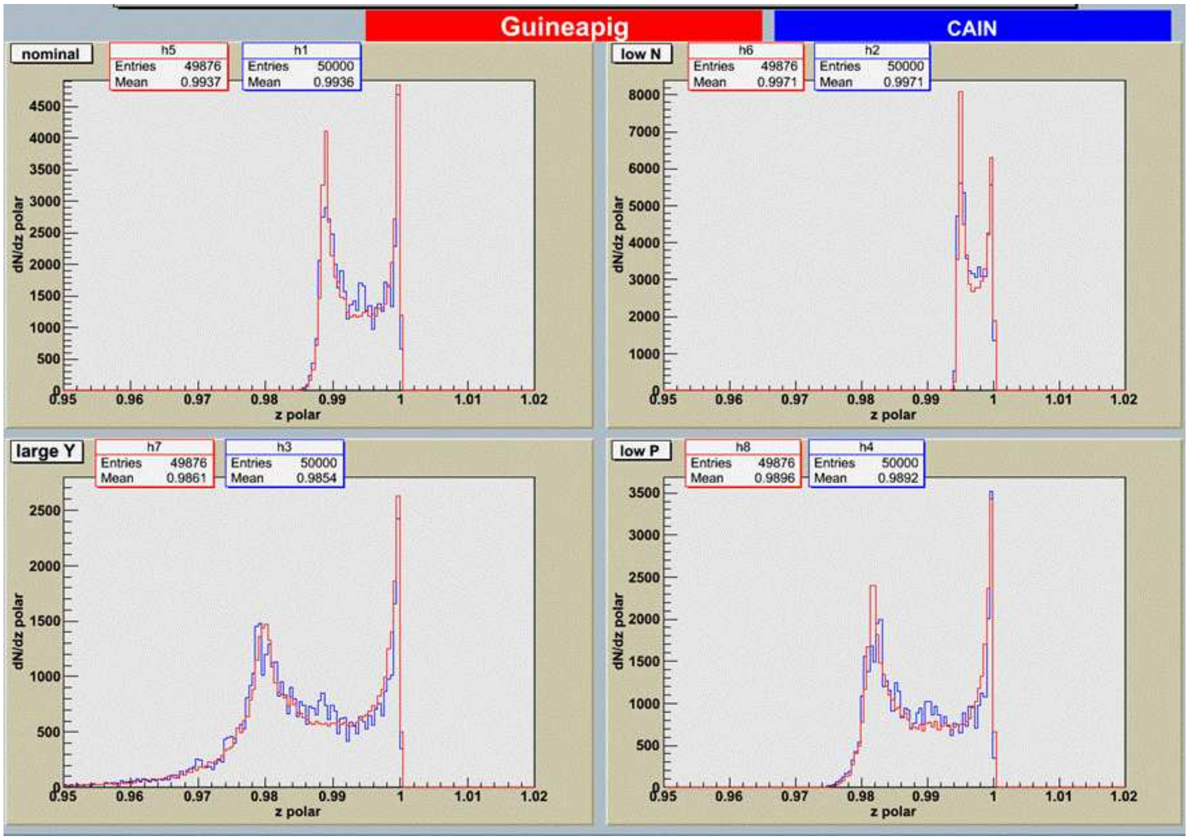


Figure 1: Comparison between GP++ and CAIN simulations for the T-BMT process for the four beam parameter sets of the ILC at 500 GeV center-of-mass energy.

| Total $\Delta P$ [%] | Nominal | Low N | Large Y | Low P |
|---|---|---|---|---|
| GP++ | 0.72 | 0.36 | 1.48 | 1.24 |
| CAIN | 0.74 | 0.36 | 1.53 | 1.26 |
| CAIN-GP++/CAIN [%] | 2.70 | 0 | 3.27 | 1.59 |

Table 3: Total depolarization after a full beam-beam interaction, for the four beam parameter sets of the ILC at 500 GeV center-of-mass energy.

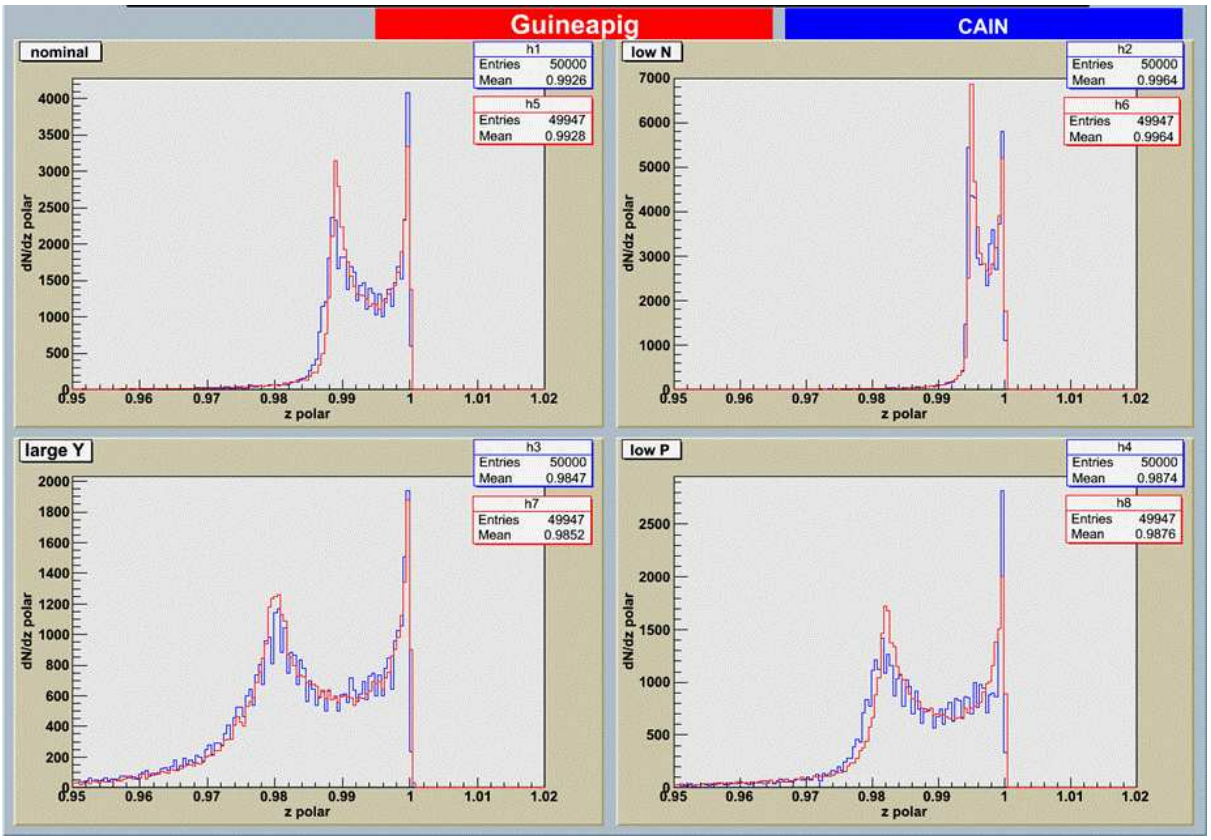


Figure 2: Comparison between GP++ and CAIN simulations for the total depolarization for the four beam parameter sets of the ILC at 500 GeV center-of-mass energy.

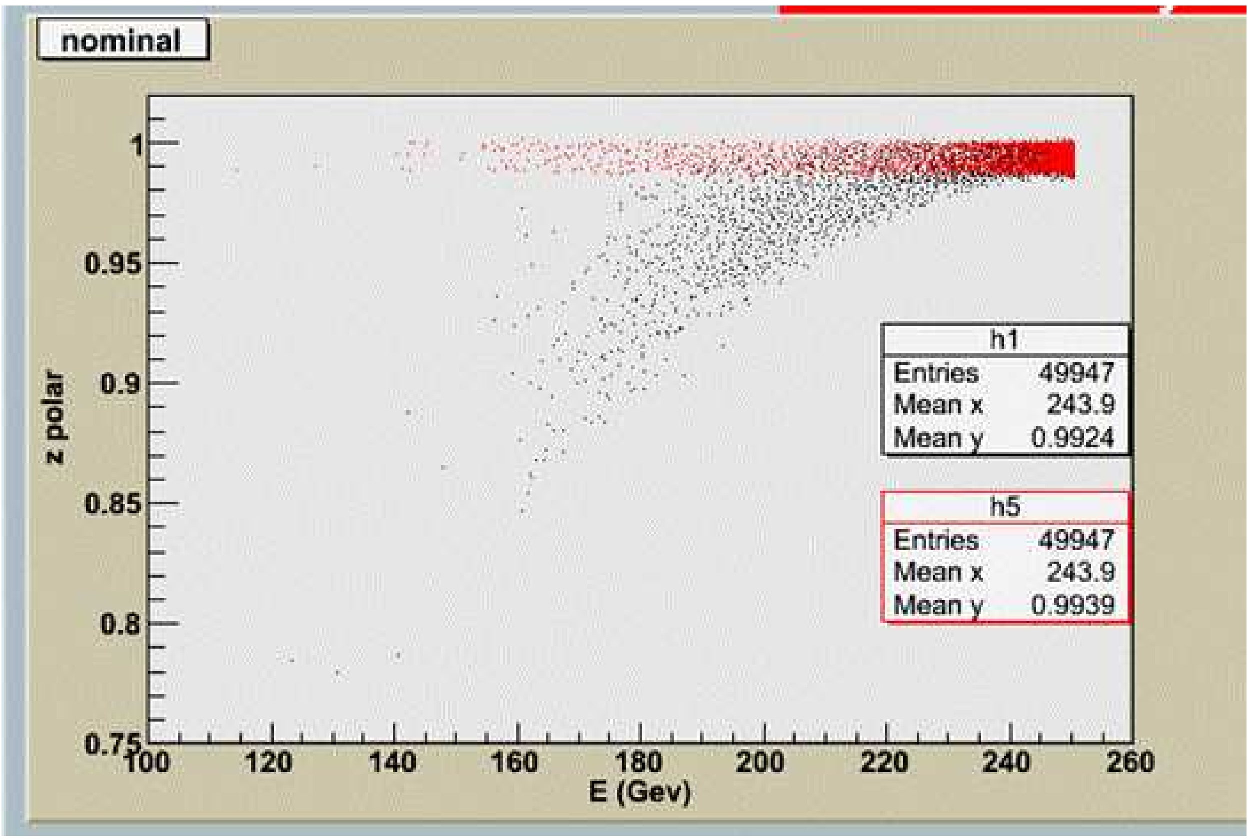


Figure 3: Dependance of the depolarization with the energy of the macro-particles, for the T-BMT precession only (in red) and including also the spin-flip mechanism (in black).

# 4 Spin state of luminosity contributing beam particles

For physics, it is essential to know the spin state of the particles which interact at the quantum level. This can be obtained computing the polarization of the beam particles making up the luminosity which is calculated in the simulation, also referred to as luminosity-weighted polarization, $P_{lw}$.
GP++ provides the polarization state, $P_x, P_y, P_z$ of macro-particles at collision time, whereas CAIN provides specific luminosities corresponding to the possible sets of spin states of the two beams, $L_{zz}, L_{zx}, L_{zy}$.... Starting from 100% initial polarizations, $P = (0, 0, 1)$, the luminosity-weighted depolarization, $\Delta P_{lw}$, is calculated as $1 - \sqrt{P_{z1} P_{z2}}$ in GP++, while in CAIN, it is computed as $1 - \sqrt{L_{zz}/L}$.
Tables 4 and 5 show the comparisons for the luminosity-weighted depolarizations computed in the two simulations taking into account, respectively, precession only and both precession and spin-flip mechanisms.
Agreement better than 95% is obtained for the Nominal and Low charge (Low N) ILC beam parameter sets. For the large disruption (Large Y) and Low Power options, discrepancies of up to 20% are however found.
The contribution from spin-flip effects amounts to between 10% and 40% of the total depolarization. Though it can be expected to depend on beam parameters, the 0.273 ratio of luminosity-weighted to total depolarizations which was estimated analytically in [3, 4] is well reproduced by the GP++ simulation.

| $\Delta P_{lw}$ [%] from precession | Nominal | Low N | Large Y | Low P |
|---|---|---|---|---|
| GP++ | 0.17 | 0.08 | 0.41 | 0.28 |
| CAIN | 0.19 | 0.09 | 0.48 | 0.30 |
| CAIN-GP++/CAIN [%] | 10 | 11 | 15 | 7 |

Table 4: Luminosity-weighted depolarization due to precession for the four 500 GeV center-of-mass energy beam parameter sets of the ILC.

| Total $\Delta P_{lw}$ [%] | Nominal | Low N | Large Y | Low P |
|---|---|---|---|---|
| GP++ | 0.23 | 0.13 | 0.46 | 0.41 |
| CAIN | 0.24 | 0.13 | 0.57 | 0.50 |
| CAIN-GP++/CAIN [%] | 4 | 0 | 19 | 18 |

Table 5: Total luminosity-weighted depolarization for the four 500 GeV center-of-mass energy beam parameter sets of the ILC.

# 5 Dependance with beam parameters

The dependance of the luminosity-weighted depolarization with beam parameter variations was also tested with GP++.

As displayed in Figure 4, no drastic dependances are observed for horizontal or vertical beam offsets in ranges for which a reasonable luminosity is maintained. For a vertical beam offset of two sigma, which is very pessimistic, an absolute variation of only 0.03% is found for the luminosity-weighted depolarization. The dependance with vertical beam size also shows good stability for variations of up to 50%.

There is however a large sensitivity to changes in the horizontal beam size. In Figure 5 it is shown that a 10% variation would lead to an absolute uncertainty of 0.05% on the depolarization, representing a 20% relative error.

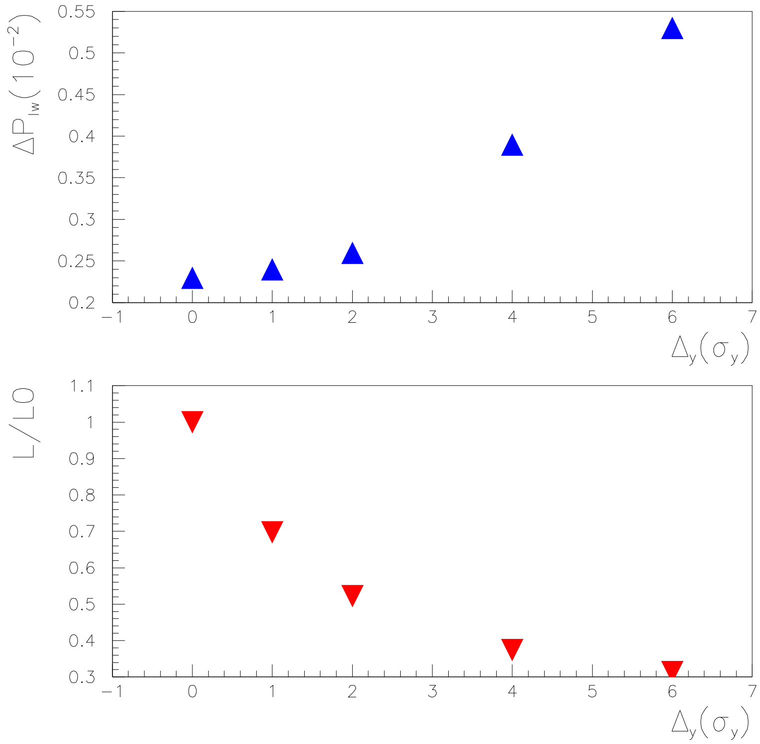


Figure 4: Dependance of the luminosity-weighted depolarization (upper) and of the luminosity (lower) on the vertical offset for the Nominal ILC beam parameter set at 500 GeV center-of-mass energy.

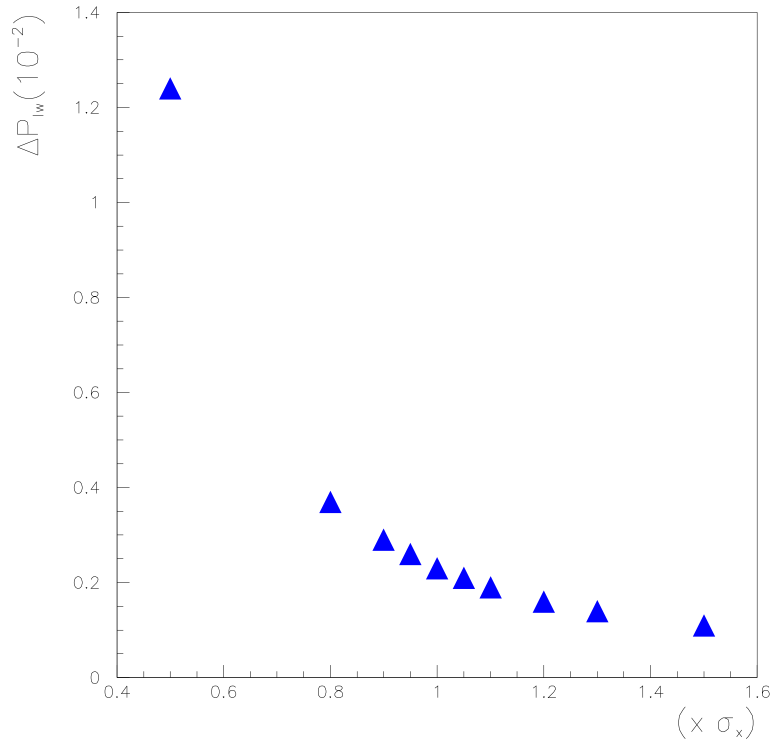


Figure 5: Dependance of the luminosity-weighted depolarization on the horizontal beam size for the Nominal ILC beam parameter set at 500 GeV center-of-mass energy.

# 6 Conclusion

A spin dynamics treatment including precession and spin-flip effects has been implemented in GP++, following that already available in the CAIN simulation. Studies of the depolarizing effects induced by the beam-beam interaction using this new simulation show both reasonable qualitative and quantitative behaviour.
Explicit comparisons with the CAIN simulation are presently the only way to test this new development in GP++. Although some differences are found, good overall agreement is obtained for the main ILC beam parameter sets.
For Nominal ILC parameters at 500 GeV, a luminosity-weighted depolarization of $0.23 \pm 0.01\%$ is predicted with both simulations, confirming prior studies using CAIN[6].
A first study of the dependance with beam parameters indicates that the horizontal beam size must be controled to better than 10% to maintain a relative precision of better than 20% on the degree of polarization.

# 7 Acknowledgement

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