



## TOWARDS SUB-MICROMETER RESOLUTION OF SINGLE SHOT STRIP LINE BPM

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### Abstract

A high resolution single shot BPM set-up is designed. One of the BPM modifications is developed as a strip-line-pickup-based difference-sum BPM. In this BPM, each strip line signal is converted into a three-700MHz-wave burst in a time domain converter. The difference and sum bursts produced by a hybrid junction are detected in a pair of synchronous detectors. The synchronous detector reference signals, and ADCs clock triggers are manufactured from the sum burst. The set-up and features of this BPM are presented. BPM resolution estimation is done, contributions of various noise components are considered. A method is proposed to measure components individually. The BPM prototype resolution was measured using a KEK ATF beam. For bunch intensity range  $\geq 0.3\text{nC}$  the resolution is about  $1\mu\text{m}$  (for BPM effective aperture 1/5). Resolution measurement results are discussed. With appropriate ADCs, the BPM can measure individual bunches at a rate of up to 50MHz. The BPM latency to the ADC inputs is as low as 10ns. High resolution and low latency together, make this BPM suitable for fast feedback/feed-forward systems.

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## INTRODUCTION

In this paper, some results of the development of a single shot BPM, based on an electrode pickup, are described. The development of this device was driven by challenging beam diagnostics requirements for the ILC, ATF2 [1, 2], and the EMMA [3] accelerators.

The BPM should meet the following requirements:

- Bunch charge: 1nC (ATF2); 0.01– 0.1nC (EMMA).
- Bunch spacing (getting-ready-to-the-next-bunch time): 300–150ns (ATF2); 50ns (EMMA).
- Vacuum chamber aperture: 25mm (ATF2); 40mm (EMMA).
- Beam position measurement range: a few mm (ATF2); a full aperture (EMMA).
- Pickup: strip line electrodes (ATF2); button electrodes (EMMA).
- Resolution:  $\leq 1\mu\text{m}$  (ATF2);  $\leq 50\mu\text{m}$  (EMMA, 30pC).
- Analog processing latency:  $\ll 150\text{ns}$  (ATF2).

An attempt was made to develop a processing scheme and a corresponding set of matching each other  $50\Omega$  circuits that could be combined into some ‘optimal’ single shot BPM that would meet the requirements above, in particular, which would have highest resolution in a full pickup aperture or, within a reasonably reduced position range, a resolution close to the thermal noise resolution limit. A BPM modification for the EMMA is now under development. Here we focus at a BPM modification that was meant for the ILC Crab Cavity Test Bench [1] and Feed-Forward Correction System [2] at the ILC/ATF2.

## DIFFERENCE-SUM BPM SET-UP

In the BPM, each strip line pickup signal is converted to a flat-top-envelope three-wave burst. The difference and sum bursts from the outputs of the hybrid junction are amplified and detected in a pair of synchronous detectors. The output of each detector is smoothed with a LP filter, the output pulse of which is sampled at its end flat part by an ADC.

A short burst with a flat top envelope is an important feature of the BPM. With it, three goals are attained: first, the BPM gets a short getting-ready-to-the-next-bunch time; next, it has a short latency as the filter output that is sampled at its end is of the same length as the burst; finally, with an envelope flat top and using an overshoot-less Gaussian filter, it is possible to obtain in a short burst time a BPM output pulse with a flat end. The flatness at the sample moment is significant as it enables to reduce additional noise generated by ADC clock jitter, and to relax the tolerance of sample moment positioning as well.

An initial variant of a pickup-signal-to-flat-top-envelope-burst converter [4] was based on a sine-wave irregular-strip-line coupler described in [5]. A coupler’s wave was split into four waves each of which was progressively delayed and then re-combined. The output signal is shown in Fig. 1.

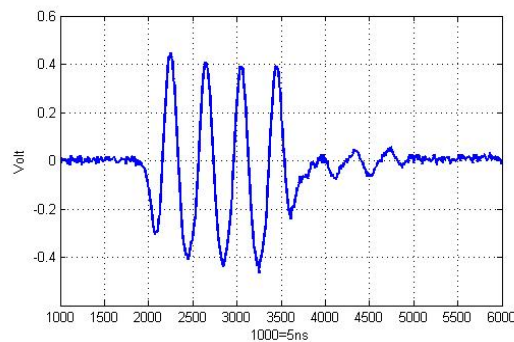


Fig.1. An output re-combined signal from a strip-line-coupler-based converter. The burst is 500MHz. A 1.5GHz oscilloscope TDS7154B. Beam intensity is about 1nC.

Later a more compact variant was developed as a  $50\Omega$  time domain converter consisting of three cascades each of which is a single-rectangular-wave coupler similar to the coupler mentioned above. The converter is made as a PCB with a pair of coupled strip lines between two ground planes. To get a flat envelope in a converter of such kind, it is necessary in the design to adjust the coupling strengths in the first two cascades to have them matched to the last cascade.

An output signal of a converter prototype (no coupling strengths adjustment is done) is shown in Fig. 2.

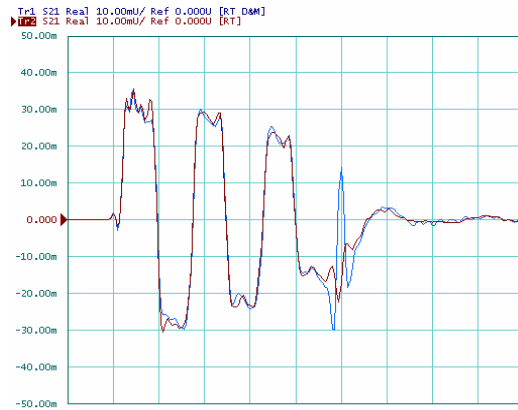


Fig.2. Delta function response (red) of a cascaded-coupler-based converter prototype. The burst is 700MHz. A Network Analyser E5071B, a Gaussian impulse 115ps.

In this converter, the wave half-period is made equal to the spacing of the impulses in the strip line pickup signal. Starting with the second half-wave in the burst, the magnitude is doubled as the waves excited by the pickup's opposite polarity impulses are summed with opposite phases. The last half-wave is similar to the first half-wave. So, the burst envelope has a trapezoidal shape. A beam signal of the mentioned above converter prototype is shown in Fig. 3.

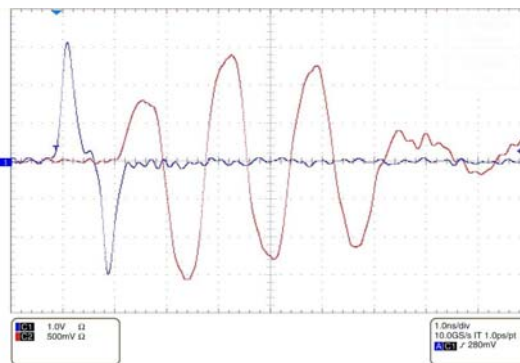


Fig.3. A cascaded-coupler-based converter prototype signal (red). The converter input signal (blue, attenuated 12dB) is a 12cm strip line pickup signal at the end of a 30m length Helix cable. A 1.5GHz oscilloscope TDS7154B. Beam intensity is about 1nC.

The BPM is based on a (0.01-2)GHz difference-sum hybrid junction H-9 from Anzac–M/A-Com. In the beam tests, it was sufficient to balance the BPM prototype input cables together with the hybrid junction simply by manual adjustment of a pair of input phase shifters and watching the difference signal on oscilloscope. A problem of BPM zero offset stability and zero offset (remote) adjustment methods and procedures are left beyond the scope of this paper.

A 20dB-gain 1.8GHz-bandwidth 1.5dB-noise-factor amplifier was used based on a chip RF2360 from RFMD.

As a synchronous detector, a +17dBm-LO (0.005-3)GHz mixer SYM-30DHW from Mini-Circuits was used.

The BPM output filter is made as a 50Ω diplexer with a 6-th order Gaussian LP filter and 2-nd order HP filter. The LP bandwidth is 150MHz, the 0.1-0.9 rise time is 2.2ns.

A BPM output buffer amplifier (based on a chip AD8000 from Analog Devices) has gain +12dB into 50Ω and a referred-to-input noise about 35μV in the bandwidth 150MHz.

The BPM analog outputs are measured with a pair of ADCs. Bunch intensity normalisation is supposed to be done in a digital post-processor. Sum of a BPM analog processing time, an ADC measurement time and a normalisation time amounts to a total BPM latency. The BPM processing time is about 10ns. With a commercial pipeline 100MHz ADC, its output is available after 3 (ADS5424) or 4 (AD6645) clock cycles. So, the BPM latency without normalisation equates to 40–50ns.

In this BPM, an attempt was made to make it autonomous. It has an internal beam-based synchronisation. The synchronous detector reference signal is produced by an internal circuit from the sum burst as a synchronous burst of regular amplitude. Another circuit triggered by the synchronous burst generates the ADC clock.

## DIFFERENCE-SUM BPM RESOLUTION

**I.** For the signal magnitudes  $V_d$ ,  $V_s$  at the difference-sum junction outputs the beam position  $Y$  is calculated as

$$Y = R_{\text{eff}} (g_s / g_d) (V_d / V_s) \quad (1)$$

where  $R_{\text{eff}}$  is the effective pickup radius,  $g_d$  and  $g_s$  are the junction voltage gains for the matching output loads  $r$ . For the junction output thermal noise voltage  $\sqrt{\langle u^2 \rangle} = \sqrt{4kTB \cdot (r/2)}$  in the bandwidth  $B$  the resolution can be written as

$$\sqrt{\langle y^2 \rangle} |_{\text{limit}} = R_{\text{eff}} \frac{g_s}{g_d} \cdot \frac{\sqrt{\langle u^2 \rangle}}{\bar{V}_s} \sqrt{1 + \left( \frac{\bar{Y}}{R_{\text{eff}} (g_s / g_d)} \right)^2} \quad (2)$$

where  $\bar{V}_s$  is mean rms value and  $\bar{Y}$  is mean value. The expression (2) gives a **BPM resolution lower limit** for a position measurement range  $R_{\text{eff}} (g_s / g_d)$ . The limit is inversely proportional to beam intensity and grows with beam offset (for  $g_d = g_s$ , up to  $\sqrt{2}$  times for  $\bar{Y} = R_{\text{eff}}$ ). Estimation of (2) for  $R_{\text{eff}} = 9\text{mm}$ ,  $\bar{V}_s = 0.7\text{V}$  (see below),  $\bar{Y} = 0$ ,  $r = 50\Omega$  and  $B = 150\text{MHz}$  ( $\sqrt{\langle u^2 \rangle} \approx 8\mu\text{V}$ ) gives  $\sqrt{\langle y^2 \rangle} |_{\text{limit}} = 100\text{nm}$ . With this limit, **the BPM ultimate range**  $\mathfrak{R}$  defined as  $\mathfrak{R} = \sqrt{\langle y^2 \rangle} |_{\text{limit}} / R_{\text{eff}} (g_s / g_d) = \sqrt{\langle u^2 \rangle} / \bar{V}_s$ , is  $\mathfrak{R} = -99\text{dB}$ .

**II.** Assume the signals  $V_d, V_s$  are amplified and then detected. The amplifier and detector gains and noise factors are respectively  $G_A, F_A$  and  $G_D, F_D$ . The thermal noise  $\sqrt{\langle y^2 \rangle}|_D$  at the detector output can be written as

$$\sqrt{\langle y^2 \rangle}|_D = R_{\text{eff}} \frac{g_s G_s}{g_d G_d} \cdot \frac{G_d \sqrt{\langle u^2 \rangle}}{V_s|_D} \sqrt{F_d + F_s \left( \frac{\bar{Y}}{R_{\text{eff}}(g_s/g_d)} \right)^2} \quad (3)$$

where each total gain is  $G = G_A \cdot G_D$  and each noise factor is  $F = F_A + (F_D - 1)/G_A^2$ .

The expression (3) represents evolution of the thermal noise. However, other kinds of noise may appear in the signal path. Consider here one of them. Assuming that for a BPM input signal as a sine wave burst, a synchronous detector is used where the switches are commutated by a square wave reference signal. Equating the detector to a multiplier and introducing a reference jitter  $|\delta t_{\text{ref}}| \ll T$ , where  $T$  is the period, one can calculate a detector voltage output  $V$  as

$$V = (2a/\pi)[1 - 2\pi^2(\delta t_{\text{ref}}^2/T^2)] \quad (4)$$

where for the reference signal magnitude taken equal to 1, the input wave amplitude is  $a \leq 1$ . The first term is the detected signal, the second term with  $\delta t_{\text{ref}}$  randomly changing from shot to shot is an additional voltage noise. This noise, first, is proportional to the input signal strength, second, it is a non-linear transformation product of the reference signal jitter. For Gaussian jitter, this noise is non-gaussian: it has a DC component, it is limited on one side with zero and on the other side has large fluctuations.

For an ‘optimal’ BPM this noise should not exceed the thermal noise. For a full aperture BPM with  $g_d = g_s$ , taking maximal beam offset as  $\bar{Y} = R_{\text{eff}}$  and the detector output range as  $\mathfrak{R} \cdot \sqrt{F_d + F_s}$  one can write:

$$\delta t_{\text{ref}}/T \leq (1/\pi) \sqrt{\mathfrak{R} \cdot \sqrt{F_d + F_s} / 2} \quad (5)$$

Estimation of (5) for  $\mathfrak{R} = (-99)\text{dB}$  and  $F_d = F_s = 1.6$  gives  $\delta t_{\text{ref}}/T \leq 1 \cdot 10^{-3}$ .

**III.** With a buffer amplifier at the detector output, the noise-to-signal range at the ADC input can be written as  $\mathfrak{R} \cdot \sqrt{F_{d\text{tot}} + F_{s\text{tot}}}$ , where each total noise factor is:

$$F = F_A + (F_D - 1)/G_A^2 + (F_{\text{Beff}} - 1)/G_A^2 G_D^2 \quad (6)$$

with a buffer effective noise factor taken as  $F_{\text{Beff}}$ . For an ‘optimal’ ADC, its resolution defined as  $R = \alpha_1 \alpha_2 / (2^n - 1)$  where  $n$  is the number of bits, and  $\alpha_1 \geq 1, \alpha_2 \geq 1$  are some coefficients, should satisfy the condition

$$\alpha_1 \alpha_2 / (2^n - 1) \leq \mathfrak{R} \sqrt{F_{d\text{tot}} + F_{s\text{tot}}} \quad (7)$$

For  $\mathfrak{R} = (-99)\text{dB}$ ,  $F_{d\text{tot}} = F_{s\text{tot}} = 2.5$ , and  $\alpha_1 = \alpha_2 = 1$  the number of bits is  $n \geq 16$ .

The coefficient  $\alpha_1$  is introduced to take into account the ADC transition noise. The coefficient is a rms quantity expressed with the LSB units. The coefficient  $\alpha_2$  is used to take into account the ADC clock jitter. For a flat top BPM output pulse the coefficient is  $\alpha_2 = 1$ . For a simplest case of linear slope  $V_{\text{out}} = V_0[1 + \xi(t/mT)]$ ,  $|\xi| \ll 1, 0 \leq t \leq mT$  where  $mT$  is the burst length, an acceptable clock jitter  $\delta t_{\text{clk}}/T$  in a full aperture BPM is

$$\delta t_{\text{clk}}/T \leq (\alpha_1/2^n)(2A/|V_0|)(m/|\xi|) \quad (8)$$

where  $\pm A$  is the ADC voltage range. For  $|V_0| \approx A$ ,  $\alpha_1 = 1$ ,  $n = 16$ ,  $m = 3$  and  $|\xi| = 0.1$ ,  $\delta t_{\text{clk}} / T \leq 1 \cdot 10^{-3}$ . With  $\delta t_{\text{clk}} / T = 1 \cdot 10^{-3}$ , the coefficient  $\alpha_2 = \sqrt{2}$ .

For a cupola-like top and with a sample moment at its apex, the voltage noise generated by the ADC clock jitter is a non-linear transformation of the latter and has features analogous to the features of the noise generated by synchronous detector reference jitter.

**IV.** To reduce the contribution of the additional voltage noise components generated in the detector and the ADC (and contribution of the buffer amplifier noise as well) and to come closer to the BPM resolution lower limit, a common measure is to increase the difference channel gain with reference to the sum channel gain:  $G_{\text{As}} = G$ ,  $G_{\text{Ad}} = K \cdot G$ ,  $K > 1$ . Assume that in the BPM  $g_d = g_s$ . Using the signals at the amplifier outputs, the beam position  $Y$  can be written as

$$Y = (R_{\text{eff}} / K) \cdot (V_{\text{Ad}} / V_{\text{As}}) \quad (9)$$

Assume that the input voltage ranges of the detectors in both channels are equal which can be written as  $V_{\text{Ad}}|_{\text{max}} = V_{\text{As}}|_{\text{max}}$ . In this case the measurement range  $Y_{\text{max}}$  of beam offsets shrinks  $K$  times with regard to a full aperture BPM:  $Y_{\text{max}} = R_{\text{eff}} / K$ . Using (3), the thermal noise can be written as

$$\sqrt{\langle y^2 \rangle}|_A = R_{\text{eff}} \frac{1}{K} \cdot \frac{K \sqrt{\langle u^2 \rangle}}{\bar{V}_s|_A} \sqrt{F_{\text{Ad}K} + F_{\text{As}} \left( \frac{\bar{Y}}{R_{\text{eff}}} \right)^2} \quad (10)$$

With  $Y_{\text{max}} = R_{\text{eff}} / K$  and  $F_{\text{As}} \sim F_{\text{Ad}}$  the second term under the square root can be neglected. The thermal noise magnitude at the detector input is  $K$  times enlarged, which is equivalent to having contribution of the noise in the following circuits reduced by the same factor.

The BPM output thermal noise (10) can be written as

$$\sqrt{\langle y^2 \rangle}|_{\text{out}} = R_{\text{eff}} \cdot \frac{\sqrt{\langle u^2 \rangle}}{\bar{V}_s|_{\text{out}} / G_{\text{s tot}}} \sqrt{F_{\text{d tot}K}} \quad (11)$$

where  $G_{\text{s tot}} = \bar{V}_s / \bar{V}_s|_{\text{out}}$ , and the noise factor given by the expression (6) is:  $F_{\text{d tot}K} = F_A + (F_D - 1) / K^2 G^2 + (F_{\text{B eff}} - 1) / K^2 G^2 G_D^2$ .

**V.** Consider a method that allows a measurement of the output thermal noise  $\sqrt{\langle y^2 \rangle}|_{\text{out}}$  and the output total additional noise  $\sqrt{\langle \eta^2 \rangle}|_{\text{out}}$  individually. Assuming that the additional noise as well as the thermal noise is Gaussian, the equation system for two gains  $K_i \cdot G$ ,  $i = 1, 2$  can be written:

$$\begin{cases} \langle y^2 \rangle|_{\text{out}1} + \langle \eta^2 \rangle|_{\text{out}1} = \langle r^2 \rangle|_{\text{out}1} \\ \langle y^2 \rangle|_{\text{out}2} + \langle \eta^2 \rangle|_{\text{out}2} = \langle r^2 \rangle|_{\text{out}2} \end{cases} \quad (13)$$

where  $\sqrt{\langle r^2 \rangle}|_{\text{out}i}$  are a pair of measured BPM resolution values. Write for each gain:

$$\left\{ \begin{array}{l} \sqrt{\langle r^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K_i} \cdot \frac{\sqrt{\langle V^2 \rangle}_{\text{out } i}}{\bar{V}_{\text{s out}}} \\ \sqrt{\langle y^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K_i} \cdot \frac{\sqrt{\langle u^2 \rangle}_{\text{out } i}}{\bar{V}_{\text{s out}}} = R_{\text{eff}} \cdot \frac{\sqrt{\langle u^2 \rangle}}{\bar{V}_{\text{s}}} \sqrt{F_{\text{d tot } K_i}}, \\ \sqrt{\langle \eta^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K_i} \cdot \frac{\sqrt{\langle \nu^2 \rangle}_{\text{out}}}{\bar{V}_{\text{s out}}}, \end{array} \right. \quad (14)$$

where  $\sqrt{\langle V^2 \rangle}_{\text{out } i}$  is the difference channel total voltage noise,  $\sqrt{\langle \nu^2 \rangle}_{\text{out}}$  is the additional noise expressed also in Volt. Note that the latter does not depend on  $K$ .

Comparing values in each pair for  $i = 1, 2$  in (14), write:

$$\left\{ \begin{array}{l} \sqrt{\langle y^2 \rangle}_{\text{out } 2} = \sqrt{\langle y^2 \rangle}_{\text{out } 1} \cdot \sqrt{\frac{F_{\text{d tot } K_1}}{F_{\text{d tot } K_2}}} \\ \sqrt{\langle \eta^2 \rangle}_{\text{out } 2} = \sqrt{\langle \eta^2 \rangle}_{\text{out } 1} \cdot \frac{K_1}{K_2} \end{array} \right. \quad (15)$$

Re-write the system (13) (index out is omitted):

$$\left\{ \begin{array}{l} \langle y^2 \rangle_1 + \langle \eta^2 \rangle_1 = \langle r^2 \rangle_1 \\ \frac{F_{\text{d tot } K_1}}{F_{\text{d tot } K_2}} \cdot \langle y^2 \rangle_1 + \left(\frac{K_1}{K_2}\right)^2 \cdot \langle \eta^2 \rangle_1 = \langle r^2 \rangle_2 \end{array} \right. \quad (16)$$

The system (16) together with relations (15) allows the BPM thermal noise and the BPM additional noise to be found for each  $K_i$ ,  $i = 1, 2$ .

This method can be extended for estimation of the components of the BPM additional noise, namely, the noise due to synchronous detector reference jitter, and the ADC noise, under the same assumption of being both Gaussian. Again two BPM resolution measurements should be done, now for two difference channel output buffer amplifier gains  $G_{\text{Bs}} = G_{\text{B}}$ ,  $G_{\text{Bd } i} = k_i \cdot G_{\text{B}}$ ,  $i = 1, 2$ . For some  $K$  write:

$$\left\{ \begin{array}{l} \sqrt{\langle r^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K \cdot k_i} \cdot \frac{\sqrt{\langle V^2 \rangle}_{\text{out } i}}{\bar{V}_{\text{s out}}} \\ \sqrt{\langle y^2 \rangle}_{\text{out } i} = R_{\text{eff}} \cdot \frac{\sqrt{\langle u^2 \rangle}}{\bar{V}_{\text{s}}} \sqrt{F_{\text{d tot } K}} \\ \sqrt{\langle j^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K \cdot k_i} \cdot \frac{k_i \cdot \sqrt{\langle \nu^2 \rangle}}{\bar{V}_{\text{s out}}} \\ \sqrt{\langle \rho^2 \rangle}_{\text{out } i} = R_{\text{eff}} \frac{1}{K \cdot k_i} \cdot \frac{\sqrt{\langle \mathcal{G}^2 \rangle}}{\bar{V}_{\text{s out}}} \end{array} \right. \quad (17)$$

where  $\sqrt{\langle y^2 \rangle}_{\text{out } 1} = \sqrt{\langle y^2 \rangle}_{\text{out } 2} = \sqrt{\langle y^2 \rangle}_{\text{out}}$  is the thermal noise,  $\sqrt{\langle j^2 \rangle}_{\text{out } 1} = \sqrt{\langle j^2 \rangle}_{\text{out } 2} = \sqrt{\langle j^2 \rangle}_{\text{out}}$  is the synchronous detector noise at the BPM output,  $\sqrt{\langle \nu^2 \rangle}$

is the detector voltage noise at its output,  $\sqrt{\langle \rho^2 \rangle}_{\text{out } i}$  is the ADC noise,  $\sqrt{\langle g^2 \rangle}$  is the ADC noise expressed in Volt.

The equation system reads (index out is omitted):

$$\begin{cases} \langle y^2 \rangle + \langle j^2 \rangle + \langle \rho^2 \rangle \Big|_1 = \langle r^2 \rangle \Big|_1 \\ \langle y^2 \rangle + \langle j^2 \rangle + \left(\frac{k_1}{k_2}\right)^2 \cdot \langle \rho^2 \rangle \Big|_1 = \langle r^2 \rangle \Big|_2 \end{cases} \quad (18)$$

For  $K$  given and the thermal noise  $\sqrt{\langle y^2 \rangle}$  known (that can be measured by the method (16) and (15) above), the system (18) together with relations (17) allows the synchronous detector noise at the BPM output and the ADC noise to be found.

## BEAM-BASED RESOLUTION TEST

A summary of the BPM resolution test results was presented in [6]. Below the results are re-considered and discussed.

BPM resolution tests were done at the KEK ATF. Two beam runs were used. In the first run, a single BPM was used. Its inputs were fed with the signals obtained by splitting a signal of a single pickup strip line. The differential signal was close to zero and did not change with beam displacement. The test was done for a fixed difference channel gain value  $G_d = G$  and two lower sum channel gain values  $G_{s_i} = G / K_i$ ,  $K_i > 1$ ,  $i = 1, 2$ .

In the second run (three months later), two BPMs were used connected in parallel (through splitters) to a pickup strip line pair. The differential signals were corresponding to about 0.3mm, which included a beam offset as well as a BPM zero offset. The sum channel gain was  $G_s = G / K_2$ .

The BPM prototype was calibrated against an ATF Extraction Line BPM, using a dipole steering coil of known strength. The pickup effective radius was measured as  $R_{\text{eff}} \approx 9\text{mm}$ .

The bunch intensity was about 0.8nC ( $5 \cdot 10^9 e$ ). For a hybrid junction with the inputs fed directly from splitter outputs, the sum output burst envelope rms value was  $\bar{V}_s \approx 0.7\text{V}$ .

**I.** In the first run, the strip line pickup signals were attenuated by (-7)dB for gain value  $G_{s_1}$  and by (-4)dB for  $G_{s_2}$ . That was equivalent to a virtual decrease of the run beam intensity by up to  $I_1 = 0.36\text{nC}$  and  $I_2 = 0.50\text{nC}$ , and a decrease of the sum hybrid junction output  $\bar{V}_s$  by up to  $\bar{V}_{s_1} = 0.36\text{V}$  and  $\bar{V}_{s_2} = 0.50\text{V}$  respectively.

To get the BPM sum output  $\bar{V}_s|_{\text{out}}$  well within dynamic range (about (-1.4)V against a pedestal +0.975V set at the buffer amplifier output), the gains were set to  $G_{s_1} = G / 3.16$  (-10dB) and  $G_{s_2} = G / 5.0$  (-14dB). So, the difference channel gain exceeded the sum channel gain  $K_1 = 3.16$  and  $K_2 = 5.0$  times respectively.

For  $G_{s_i}$  given, at a shot of number  $p$ , the bunch 'position' was calculated as

$$r_{i p} = R_{\text{eff}} \frac{1}{K_i} \cdot \frac{V_{\text{d out } p}}{V_{\text{s out } i p}} \quad (19)$$

The BPM resolution was calculated as a standard deviation  $std_i$  across the array (19).

Using (19), write:

$$\sqrt{\langle r^2 \rangle}_i = R_{\text{eff}} \frac{1}{K_i} \cdot \frac{\sqrt{\langle V_{\text{dout}}^2 \rangle}}{\bar{V}_{s_i} \cdot (G/K_i)} \quad (20)$$

Ratio of measured standard deviations  $std_1$  and  $std_2$  was expected to be equal to the ratio:

$$\frac{\sqrt{\langle r^2 \rangle}_1}{\sqrt{\langle r^2 \rangle}_2} = \frac{\bar{V}_{s_2}}{\bar{V}_{s_1}} = \frac{I_2}{I_1} = 1.4 \quad (21)$$

Write the BPM output thermal noise:

$$\sqrt{\langle y^2 \rangle}_{\text{out}} = R_{\text{eff}} \cdot \frac{\sqrt{\langle u^2 \rangle}}{\bar{V}_s} \sqrt{F_{\text{d tot}}} \quad (20)$$

With  $R_{\text{eff}} = 9\text{mm}$  and  $F_{\text{d tot}} = 2.5$ , the expression (20) yields  $\sqrt{\langle y^2 \rangle}_{\text{out1}} = 0.32\mu\text{m}$  for the intensity  $I_1 = 0.36\text{nC}$  and  $\sqrt{\langle y^2 \rangle}_{\text{out2}} = 0.23\mu\text{m}$  for  $I_2 = 0.50\text{nC}$ . The BPM range comes to  $\tilde{\mathfrak{R}}_1 = -89\text{dB}$  and  $\tilde{\mathfrak{R}}_2 = -92\text{dB}$ .

**II.** The BPM output signals were measured with a two channel 2GHz 14bit  $\pm 1\text{V}$  recorder GFT6004 (from Acquiretek). Assume that for inputs  $\leq |0.975 - 1.4| \text{V}$  its effective resolution is  $1/(13\text{bit}) = -78\text{dB}$ . For  $K_1 = 10\text{dB}$  the effective resolution comes to  $R_1 = -88\text{dB}$ . For  $K_2 = -14\text{dB}$  the effective resolution is  $R_2 = -92\text{dB}$ . Assume here that the recorder has a transition noise only which is Gaussian and is equal to the effective resolution above. Now the effective transition noise can be calculated as  $\sqrt{\langle \rho^2 \rangle}_{\text{out}} = (R/\tilde{\mathfrak{R}}) \cdot \sqrt{\langle y^2 \rangle}_{\text{out}}$ , which yields  $\sqrt{\langle \rho^2 \rangle}_{\text{out1}} = 0.36\mu\text{m}$ ,  $\sqrt{\langle \rho^2 \rangle}_{\text{out2}} = 0.23\mu\text{m}$ . Assuming that the synchronous detector noise is zero, one can write **a lower limit of expected BPM resolution** as  $\sqrt{\langle r^2 \rangle}_{\text{limit}} = \sqrt{\langle y^2 \rangle + \langle \rho^2 \rangle}_{\text{out}}$ , which yields  $\sqrt{\langle r^2 \rangle}_{\text{limit1}} = 0.48\mu\text{m}$  and  $\sqrt{\langle r^2 \rangle}_{\text{limit2}} = 0.33\mu\text{m}$ .

The recorder was free-running with 2GHz sampling rate and got stopped (with some delay) at each shot by the BPM sum signal. As its clock was unsynchronised with beam, the BPM pulse magnitude at each shot was calculated as a maximum of a polynomial interpolation curve constructed on a maximum sample and two adjacent samples.

**III.** For the first run a position reading std across a shot array gave the BPM resolution. The std values for  $K_1 = 10\text{dB}$  are given in Table 1, for  $K_2 = 14\text{dB}$  are given in Table 2. The array lengths  $N$  (numbers of shots recorded) are also given.

Table 1

array	1	2	3	4	5
$N$	44	65	30	30	124
$std, \mu\text{m}$	3.8	1.3	1.8	1.5	1.6

Table 2

array	6	7	8
$N$	110	120	120
$std, \mu\text{m}$	1.0	0.8	1.1

The array 1 std has an outstanding value. This is a result of a step-like jump in the difference signal, which occurred probably due to some occasional moving of the BPM input cables. A corresponding jump in the ‘position’ (about  $5\mu\text{m}$ ) is seen in a shot-by-shot plot given in Fig. 4, a). The sum signal is given in Fig. 4, b). Below, this value is excluded from the set.

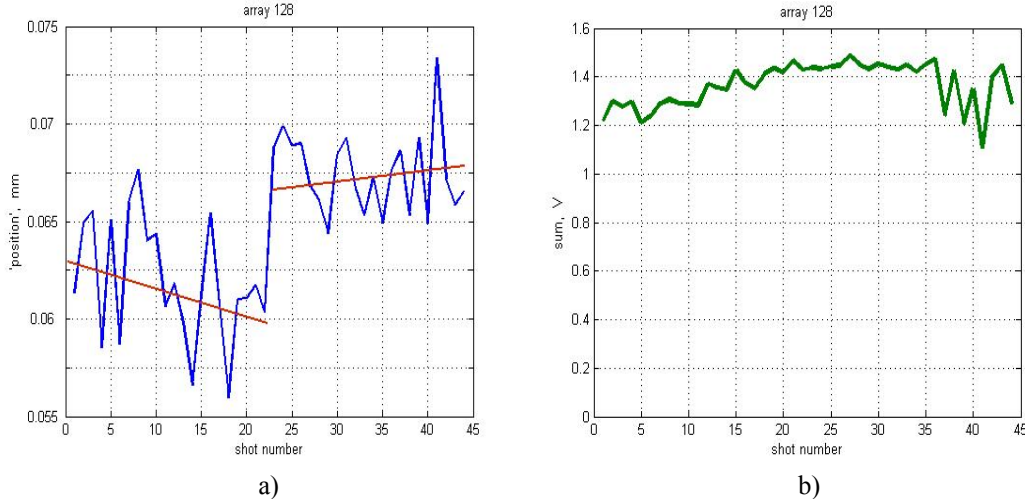


Fig.4. The ‘position’ (a) and the sum signal (b) vs shot number in the array 1.

Calculate average values  $std = \sqrt{(1/N) \cdot \sum_{p=1}^N std_p^2}$  for each Table:  $std_1 = 1.56\mu\text{m}$  and  $std_2 = 0.97\mu\text{m}$ . Ratio  $std_1 / std_2 = 1.6$  exceeds the estimate (21) above by 15%.

Each std value is about 3 times greater than the lower limit of expected BPM resolution that was estimated above as  $\sqrt{\langle r^2 \rangle}_{\text{limit1}} = 0.48\mu\text{m}$  and  $\sqrt{\langle r^2 \rangle}_{\text{limit2}} = 0.33\mu\text{m}$  respectively. For the case  $G_d = G_s = G$ , this BPM would have  $std$  close to  $5\mu\text{m}$ .

So, the additional noise of this BPM prototype turned out to be significantly larger than the thermal noise estimated above. In the second run, it was discovered that the recorder contributed some additional noise (see below). The mentioned above interpolation of the recorder readings may generate some noise as well. Noise due to synchronous detector reference jitter should also be considered.

This BPM prototype with  $K = 14\text{dB}$  has a bunch intensity range that extends down to about  $0.1\text{nC}$  provided the gain  $G$  is set in inverse proportion to intensity. For that intensity, the output thermal noise would equal the additional noise. The BPM beam offset measurement range is  $1/K = 1/5$  of the effective pickup aperture.

**IV.** For the second run, the BPM resolution was calculated as a divided-by- $\sqrt{2}$  std across a position difference array obtained from the readings of the two BPMs. A std value of one of the arrays is given in Table 3.

Table 3

array	9
$N$	128
$std, \mu\text{m}$	1.5

The std value is 1.5 times greater than  $std_2$  obtained in the first run for the same gain 14dB. Whilst data processing, it was discovered that some excessive noise is generated by the

recorder, and the larger is the recorder input, the larger is this noise. So, most probably, the increase of the *std* above occurred due to an increase of the excessive noise that in its turn occurred due to departure of the BPM readings from zero by up to 0.3mm.

Investigation showed that any array of recorder readings contains a parasitic signal consisting of harmonics of 100MHz, and its spectrum being irregularly changing, generally becomes broader with larger input. The parasitic signal originates probably from the recorder internal clock circuit, a master generator of which has frequency just 100MHz.

In Fig. 5, two recorder readings spectra are shown. They were measured without beam for the synchronous detector OFF. The recorder continuously measured output of the BPM buffer amplifier. The spectrum a) is for zero DC output, the spectrum b) is for DC output +0.975V (a sum channel pedestal).

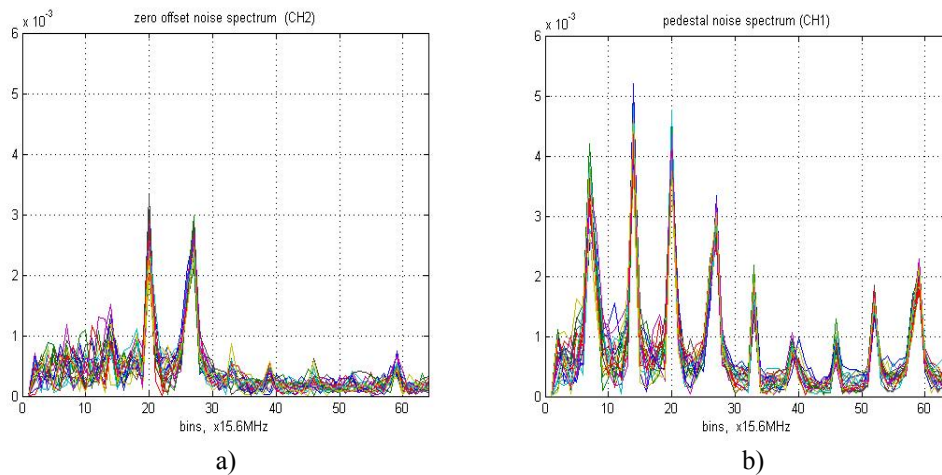


Fig.5. Spectra of recorder DC input readings.

In Fig. 5, in the range 0 to 20 bins, a buffer amplifier thermal noise can be seen. The buffer amplifier noise was within expected range, had a continuous spectrum and didn't change with DC level. This was confirmed using an oscilloscope with FFT. The amplifier output (red) and its spectrum (green) are shown in Fig. 6. As reference, a spectrum of a 100µV 300MHz sine signal is shown (both blue).

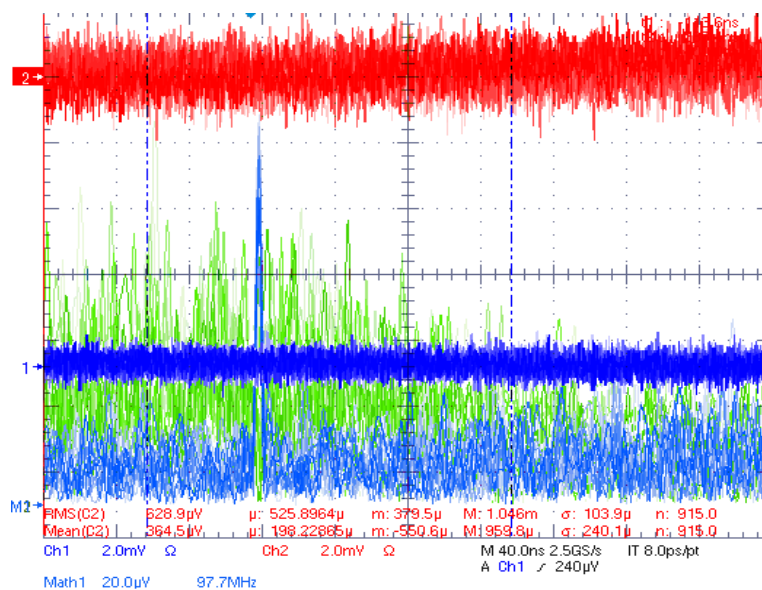


Fig.6. A spectrum of the buffer amplifier output noise.

## SUMMARY

A difference-sum BPM resolution estimation has been performed. Contributions of various noise components are considered. A method is developed to measure individually the BPM output thermal noise, the synchronous detector noise, and the ADC noise.

A high resolution low latency single shot strip line BPM set up is described. For an electron bunch intensity range  $\geq 0.3\text{nC}$ , in a test at the ATF, a BPM prototype resolution of about  $1\mu\text{m}$  has been achieved, for an effective pickup aperture of up to  $1/5$ . The test results are discussed.

For completion of BPM resolution investigation, it is necessary to equip the BPM with its own ADCs.

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