Considerations on Short Range Wakefield for CLIC Main Beam Accelerating Structures

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Abstract

The comprehension of short range wakefields is essential for the design of CLIC. Useful tools are the Karl Bane's formulas which predict the short range wake for periodic 2D symmetry structures. The comparison of 2D computations with predicted results and the study of the range of validity of these formulas are the subjects of this note. A model for rounded iris structures is also proposed.

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1 Introduction

One of the main limitations to be taken into account in the design of accelerating structures for the main beam of CLIC is the short range wakefields. Since no damping is possible the short range wake determines the maximum charge per bunch and the minimum aperture of the structures. Short range wake has thus a strong impact on the luminosity of the collider and on the efficiency of the accelerator.
The main tools for the study of the short range wake have been provided by Karl Bane in terms of analytical formulas that correlate the wake to the geometrical parameters of a periodic array of cells. K. Bane work has been done for the NLC accelerating structures. The validation of these formulas for a different geometrical range "CLIC range" is the first target of the present work. A study of the effect of rounded irises instead of rectangular ones is also part of this report.

1.1 High frequency longitudinal impedance and short range wake-field

For a periodic array of cavities according to Yokoya and Bane [1,2] the high frequency impedence (so for large wave number k) is approximately given by:

$$Z_L(k) = \frac{iZ_0}{\pi k a^2}\left[1+(1+i)\frac{\left(1-0.465\sqrt{g/p}-0.07\,g/p\right)p}{a}\sqrt{\frac{\pi}{kg}}\right]^{-1}$$

with $Z_0 = 377\Omega$, "p" the periodicity, "a" the iris aperture and "g" the cavity length (see Fig. 1).

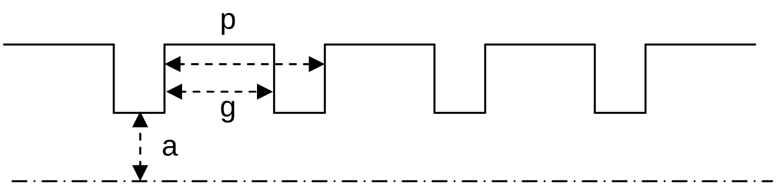

***Figure 1**: Basic shape of 2D periodic structure.*

The short-range wake is obtained by Inverse Fourier transforming:

$$W_L(s) = \frac{Z_0 c}{\pi}\exp\left(\frac{\pi s}{4 s_{00}}\right) erfc\left(\sqrt{\frac{\pi s}{4 s_{00}}}\right) \qquad (1)$$

with $s_{00} = \frac{g}{8}\left(\frac{a}{\left(1-0.465\sqrt{g/p}-0.07\,g/p\right)p}\right)^2$ for $s>0$ and $W_L(s) = 0$ for $s<0$.

For short s (1) can be rewritten in the following simpler way:

$$W_L(s) \approx \frac{Z_0 c}{\pi a^2}\exp\left(-\sqrt{\frac{s}{s_{00}}}\right) \qquad (2)$$

The relation between the longitudinal wake W_L and transverse wake W_x for small s is the following [2,3]:

$$W_x(s) = \frac{2}{a^2}\int_0^s W_l(s')ds' \qquad (3)$$

Combining (2) and (3) we get:

$$W_x(s) = \frac{4Z_0 c s_{00}}{\pi a^4}\left[1-\left(1+\sqrt{\frac{s}{s_{00}}}\right)\exp\left(-\sqrt{\frac{s}{s_{00}}}\right)\right] \quad (4)$$

The (4) is the K. Bane formula for transverse short-range wakefield; the value of S_{00} and its dependence on *a,g,p* has been modified by K. Bane by fitting with computational results and it is rappresented by the following expression [2]:

$$S_{00} = 0.169\frac{a^{1.79} g^{0.38}}{L^{1.17}} \quad (5)$$

2 Numerical computation

The short range wakefield have been compute with the 2D code ABCI [4] which solves the Maxwell equations in the time domain for axi-symmetric structures. ABCI makes use of a moving mesh which drastically reduces the number of mesh points which should be stored. The moving mesh option allows thus the computation of very short bunches. In order to evaluate the limit of the computation, several Gaussian beams have been computed with the same geometry. The ratio between the sigma and the mesh density has been kept constant for the different computations. The results show very good agreement for sigmas larger than 70 microns but also not tolerable discrepancies for short beam bunches (sigma of the order of 30-50 microns); see fig. 2. In the rest of this note all the computations have been done for sigmas of 100 microns to have reasonably correct results.
The CLIC bunch length, which is around 40-50 microns, is on the border line; for this reason is very important to have also a non-computational approach to study the short range wakefield especially to describe non Gaussian distributions.
K.B. formulas are obviously independent from the bunch length and distribution and are the best tool to describe short range wakefields.

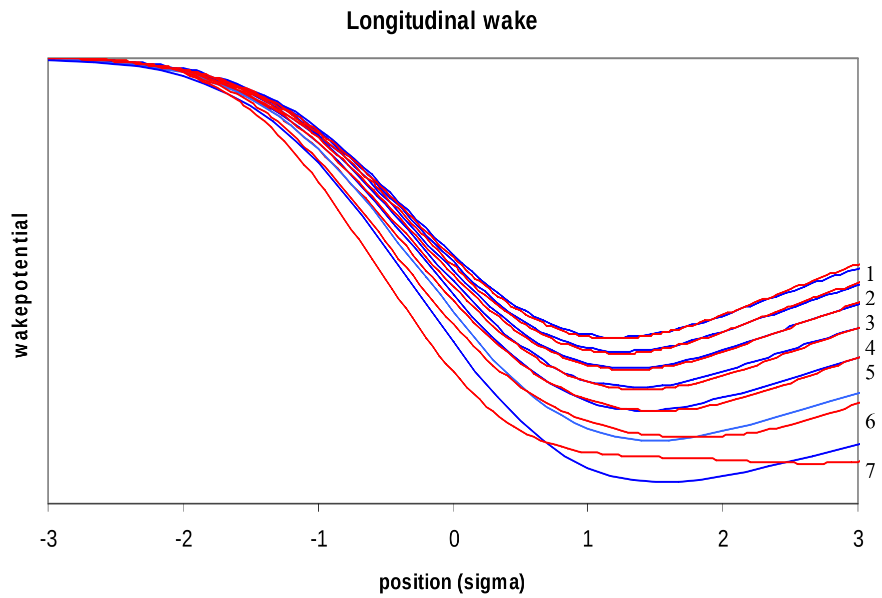

Figure 2*: ABCI (red lines) and convolutions of K. Bane (blue lines) longitudinal wake for different sigmas (1=0.15 mm, 2=0.13mm, 3=0.11 mm, 4=0.09mm, 5=0.07mm, 6=0.05mm, 7=0.03mm; for a Gaussian distribution).*

K. Bane gives for transverse wakefield (4) (5) the following geometrical range of validity: $0.34 \le a/p \le$ and $0.54 \le g/p \le 0.89$ [2].

The present CLIC study covers a much larger area in the *a/p*, *g/p* plane, for this region a check of validity of the (4) and (5) has been done; see fig. 3.

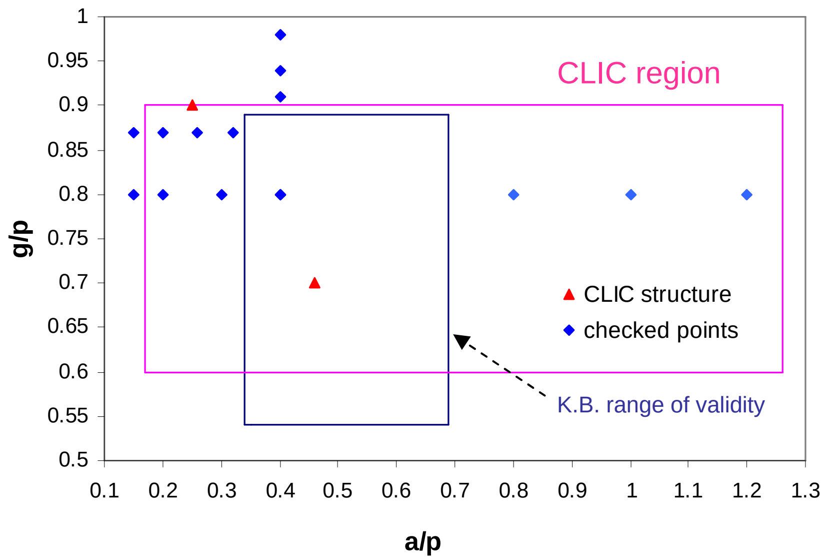

***Figure 3**: The range of validity of (4) and (5) does not cover the full CLIC range. In red are shown the first and last cell of the present CLIC reference structure.*

The computational results show that the K. Bane formula range of validity is much larger for the longitudinal wakefield and it covers the full CLIC region in the space *g/p*, *a/p*. For the transverse wakefield, computational results show good agreement with K. Bane in the full *g/p* range but also not negligible discrepancies for low value of *a/p*; see fig. 4.

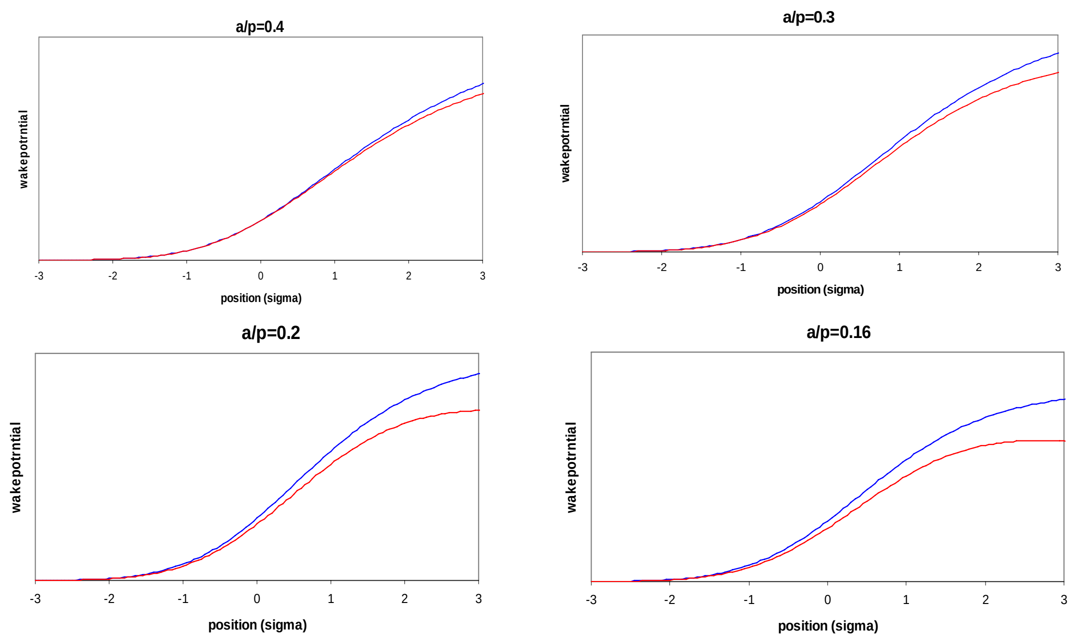

***Figure 4**: The range of validity of (4) and (5) does not cover the full CLIC range. For small aperture (a/p<0.3) the predicted values (blue) are significantly different of the computed ones (red).*

3 Validity of the K.B. for tapered structures

K. Bane formulas are valid for constant geometry, i.e. constant impedance structures; for this reasons the utilisation of these formulas for tapered structure had to be verified.

The implemented function in placet [5] is actually the following:

$$W_{tapered} = \frac{1}{4}\left[\frac{1}{2}\left(W(a+D)+W(a-D)\right)+W(a)+W\left(a+\frac{D}{2}\right)+W\left(a-\frac{D}{2}\right)\right] \qquad (8)$$

where a is the average aperture for a linearly tapered structure where D is total excursion ($D=a_{max}-a_{min}$). The expression (8) is a simplified approximation of:

$$Wx(s)=\frac{4Z_0c}{N\pi}\sum_{n=1}^{N}\frac{S_{0n}}{a_n^4}\left[1-\left(1+\sqrt{\frac{s}{S_{0n}}}\right)\exp\left(-\sqrt{\frac{s}{S_{0n}}}\right)\right] \qquad (9)$$

Where N is the total number of irises in the structure, S_{0n} is defined by the (5) with the corresponding values of a_n and g_n.

A simple case of a tapered structure composed of 28 different irises ($0.335 \le a/p \le 0.603$) has been computed and the results have been compared with the combination of the 28 convolutions of the gaussian beam with 28 expressions of the (4) (5) for the different irises.

The results show good agreement with the computations and confirm the validity of the model used in placet [5]; see fig. 5.

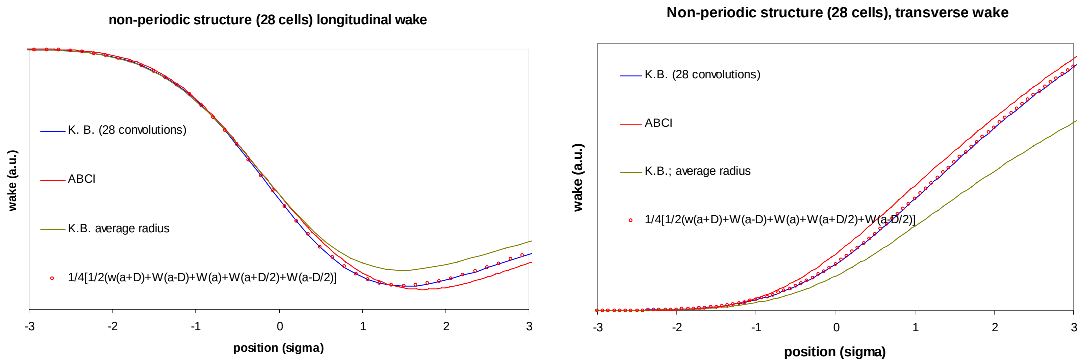

Figure 5*: Longitudinal wake (to the left) and dipole wake (to the right). The combination of the 28 convolutions is in good agreement with the computational results.*

4 Extension of the K.B. formulas to rounded iris structures

One of the main limitations of the K. Bane formulas concerns the shapes of the irises; the reference geometry is a simple 2D periodic array of squared irises. Real structures have obviously rounded or elliptical irises for evident reasons of peak electric field and machining.

Both longitudinal wake and transverse wake depend largely on the aperture a ($w_l \propto a^{-2}; W_x \propto a^{-3}$); the rounding of the iris introduces thus a significant approximation especially for the transverse wake. The modification of the model to include the rounded irises or a simple combination of different a, g, structures are two possible options to improve the estimation of the wake. The second approach is proposed.

A simple geometry (a/p=0.33, g/p=0.83) has been computed; in this case a is smallest aperture (a=2) and the radius of curvature is (p-g)/2.

The computed ABCI results of the curved irises have been compared with the composition of the convolutions of K. B. formulas of seven different a,g structures with the Gaussian beam. In the present case the geometrical values (a,g) of the seven structures have been defined by simply considering a sin-like variation of a and a cos-like variation of g for seven different angles at the regular step of 15° from 0° to 90°.

The proposed model to represent rounded irises is thus the following:

$$Wx(s)=\frac{4Z_0c}{M\pi}\sum_{m=1}^{M}\frac{S_{0m}}{a_m^4}\left[1-\left(1+\sqrt{\frac{s}{S_{0m}}}\right)\exp\left(-\sqrt{\frac{s}{S_{0m}}}\right)\right] \tag{10}$$

where $S_{0m}=0.169\frac{a_m^{1.79}g_m^{0.38}}{p^{1.17}}$; $a_m=a_0+\frac{(p_0-g_0)}{2}(1-\sin(\alpha_m))$; $g_m=g_0\cos(\alpha_m)$ and $\alpha_m=90^{\circ}\left(1-\frac{m}{M-1}\right)$ and M is a positive integer large enough (M=7 in the present case) to provide a good representation of the problem.

The results are in good agreement with the computations especially for the transverse wake; see fig. 6.

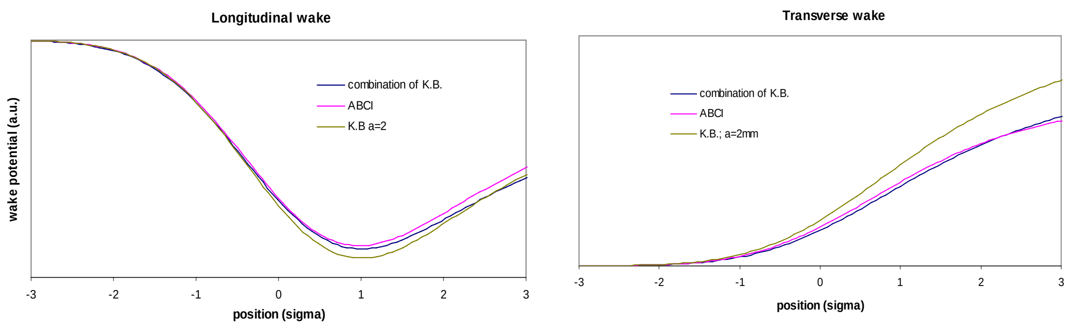

***Figure 6**: Longitudinal wake (to the left) and dipole wake (to the right).In both cases the agreement of the model expressed by the (10)(blue line) and the computational results(red line) is very good. The results show also how the conventional K.B. formulas (green line) overestimate the wakes especially the dipole one.*

The results show how the transverse wake is largely overestimated in the present case (a=2mm, a/p=0.33, g/p=0.8333 and σ= 0.1 mm). A better comprehension of the effect of rounded irises and the validation of the (10) over a large range of *a/p* and *g/p* could provide a useful tool for optimising the design of RF structures.

5 Conclusion

The range of validity of K. Bane formulas covers almost all CLIC requirements with the exception of the dipole wake for low value of *a/p* where the wake seems to be overestimated. A further approximation of K.B. formulas is due the rounding of the irises; also in this case analytical formulas tend to overestimate the wake. The (10) seems to be a promising tool to describe the effect of the rounded iris. A new fitting of the (6) for the CLIC geometrical range is under way.

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