EUROTeV-Report-2007-063

Analysis of a nearly-confocal resonator for parasitic external modes rejection

S. Cheng, A. Ferrari, A. Rydberg and E. Öjefors*

September 17, 2007

Abstract

A numerical and experimental study of a nearly-confocal resonator with two spherical mirrors is presented in this paper. The geometry was chosen in order to have a large quality factor for the diffraction losses at 12 GHz, and thereby a weak coupling to parasitic external TE and TM modes, that can propagate in a beam pipe on which the resonator may be installed. Simulations were performed with the time domain solver of the CST Microwave Studio simulation package and the frequency domain solver of Ansoft HFSS respectively. A large number of propagating modes in the beam pipe were numerically analyzed. In order to verify the theoretical studies, a prototype consisting of a nearly-confocal resonator installed onto a 30 cm long beam pipe, with a 20 cm long standard M-band waveguide connected to the upper spherical mirror, was fabricated and measured. Experimental results show a very good agreement with the simulations.

Presented at EMB07 - 4th Swedish Conference on Computational Electromagnetics - Methods and Applications, Lund (Sweden), October 18-19, 2007.

---

*All authors at Uppsala University, Sweden

1 Introduction

Open resonators with spherical mirrors were originally investigated in the early 1960's, when the first laser oscillators in the microwave and optical regimes appeared. The electromagnetic field distribution in such cavities were first theoretically analyzed by Boyd and Gordon in 1961 [1, 2] and experimentally studied in 1970 [3, 4]. One interesting property of this type of open resonators is their potentially high quality factor, especially for the diffraction losses that result from the finite size of the mirrors. These were first calculated by Fox and Li in 1961 [5]. A very readable tutorial on such resonators is given in [6] and an overview of the formulas can be found in [7]. The highest quality factor, and thereby the smallest diffraction losses, are achieved when the distance between the mirrors is close or equal to their curvature radius. This design produces the smallest possible beam diameter at the mirrors for a given cavity length. Reciprocity [8] then suggests that, as a result of their high quality factor for the diffraction losses, open resonators with spherical mirrors only weakly couple to external fields, especially close to confocality.

In turn, this should allow a significant improvement of the signal-to-noise ratio for beam diagnostic devices based on an open resonator with spherical mirrors. Indeed, beam monitors in high intensity accelerators are often perturbed by microwave fields generated by the beam itself upstream of the detection device and that propagate in the wake of the bunches. A resonator pick-up with spherical mirrors situated transversely to the direction of propagation of the beam, with a high quality factor for the diffraction losses, is likely to be almost insensitive to parasitic TE or TM fields, while keeping a significant coupling to the direct quasi-TEM fields of the beam. A conceptual design of such a beam monitor for the third CLIC (Compact Linear Collider) Test Facility CTF3 [9] was reported in [10]. Its purpose is to monitor the evolution of the 12 GHz beam harmonic during the interleaving of 3 GHz bunch trains and the subsequent multiplication of the beam frequency by a factor four in the CTF3 Combiner Ring.

In this paper, a complete analytical, numerical and experimental investigation of a nearly-confocal resonator with spherical mirrors at 12 GHz is performed, with special emphasis on the external parasitic modes rejection. In Section 2, the geometry of this resonator is described and its quality factors are estimated. The presented nearly-confocal resonator is numerically studied using both CST Microwave Studio (CST MWS) [11] and Ansoft High Frequency Structure Simulator (Ansoft HFSS) [12]. To verify the previous theoretical and numerical studies, a prototype is fabricated and experimentally characterized. All numerical and experimental results are reported in Section 3 and 4. Finally, conclusions are given in Section 5.

2 Analytical determination of the geometry and of the quality factors

A schematic layout of an open resonator with spherical mirrors is shown in Figure 1, together with the parameters that are relevant for its design.

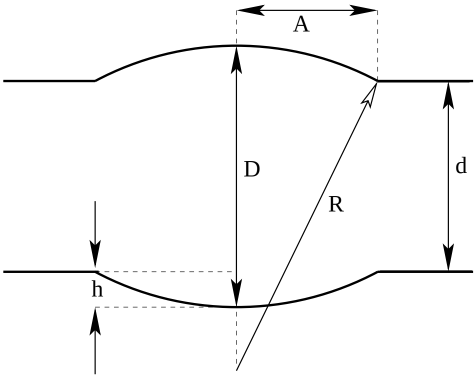

Figure 1: Schematic of a nearly-confocal resonator installed onto a rectangular pipe.

In cylindrical coordinates, the paraxial solution of the wave equation between the spherical mirrors of an open resonator is described by Gaussian beams modulated with associated Laguerre polynomials L_n^m. When solving the wave equation with vanishing fields on the mirrors, the following resonance condition is obtained:

$$f = \frac{c}{2D}\left[q + 1 + \frac{1}{\pi}\left(1 + m + 2n\right)\arccos\left(1 - \frac{D}{R}\right)\right]. \tag{1}$$

Here, q is the number of nodes between the spherical mirrors, and $(m,\, n)$ are the indices of the associated Laguerre polynomials L_n^m.

The relevant parameter to describe the diffraction losses due to the finite size of the spherical mirrors is the Fresnel number:

$$N_F = \frac{A^2}{D\lambda} \times \sqrt{\frac{2D}{R} - \left(\frac{D}{R}\right)^2}. \tag{2}$$

The fractional power loss per bounce at each mirror due to the diffraction losses can be written as follows:

$$\alpha_d = \frac{2\pi\left(8\pi N_F\right)^{1+m+2n} e^{-4\pi N_F}}{(m+n)!n!}. \tag{3}$$

The quality factor Q_d corresponding to the diffraction losses is then given by:

$$Q_d = \frac{2\pi D}{\alpha_d \lambda}. \tag{4}$$

For a given resonator geometry (and thereby a given Fresnel number), the smallest diffraction losses occur for the fundamental mode, which has $m = n = 0$.

The resistive losses, which are due to the finite conductivity at the mirror surfaces, are determined by the surface resistivity of the mirror $R_s = 1/\sigma\delta$ (where σ is the conductivity and δ is the skin depth) and by a geometry factor G, which is the same for all eigen-modes in the resonator:

$$G = Z_0 \times \frac{\pi}{2} \times \frac{D}{\lambda}, \tag{5}$$

where $Z_0 = 377\,\Omega$ is the impedance of free space.

The quality factor Q_r associated to the resistive losses is simply given by:

$$Q_r = \frac{G}{R_s}. \tag{6}$$

In this study, a mirror distance $D = 6.78$ cm and a curvature radius $R = 8.69$ cm are used. This ensures that the only eigen-mode at 12 GHz has $m = n = 0$ and $q = 4$. The vertical dimension of the pipe is $d = 3.7$ cm. The elevation of the zenith of the mirror domes above their edges and the mirror radius are $h = 1.54$ cm and $A = 4.94$ cm.

For the mode with $m = n = 0$, the quality factor associated to the diffraction losses is $Q_d = 3.6 \times 10^6$. As for the resistive losses at the surface of the Aluminium mirrors, we expect $Q_r = 4.1 \times 10^4$, which is two orders of magnitude smaller than the quality factor of the diffraction losses. In turn, reciprocity suggests that it should ensure a good reduction of the coupling to parasitic modes propagating in the pipe.

3 Numerical analysis

Each computational electromagnetic technique has its advantages and applicability bounds. In this paper, two commercial EM softwares, finite-difference time-domain method based CST MWS transient solver and finite element method based Ansoft HFSS frequency domain solver were employed in the numerical studies of the nearly-confocal resonator. The simulated results from the two different solvers were compared. In the simulations, we consider a structure that consists of two metal plates facing each other and separated by 3.7 cm, from which a spherical cavity, with a curvature radius of 8.69 cm and a depth of 1.54 cm, is carved out. A standard M-band rectangular waveguide with transverse dimensions 1.905 cm × 0.953 cm is connected to the upper mirror through the plate.

3.1 Reflection coefficient at the waveguide

A driven method was used in order to search for the eigen-modes of the nearly-confocal resonator, at and around the frequency of interest. Figure 2 shows the simulated reflection coefficient S_{11} at the upper port of the waveguide obtained from CST MWS and Ansoft HFSS. In order to allow a fine meshing of the whole structure, the resistive losses are not included and open boundaries are used for the lateral sides in the simulations. When there is no resonance, the microwave signals injected through the waveguide do not oscillate between the spherical mirrors and mostly escape through the open boundaries, so S_{11} remains rather small. On the other hand, when the resonance condition is satisfied, the diffraction losses become very small and the eigen-modes are trapped inside the nearly-confocal resonator (provided that m and thereby $m + 2n$ are even numbers in order to avoid a zero-field at the centre of the upper mirror, which would prevent from coupling to the field propagating through the waveguide). As a significant fraction of the power oscillating between the mirrors couples back to the waveguide through the large hole in the upper mirror, S_{11} becomes much closer to 0 dB. This is clearly the case at 12 GHz.

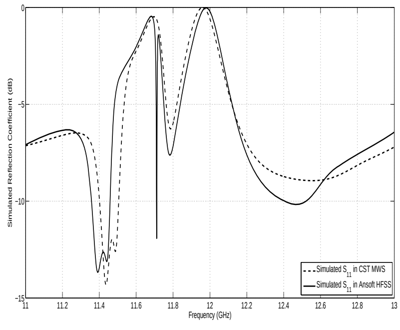

Figure 2: Simulated reflection coefficient of the nearly-confocal resonator with lateral open boundaries where the various resonances are identified with arrows.

In the frequency range of Figure 2, one expects another resonance, with both an even value of m (which ensures a non-zero field at the centre of the upper mirror) and small diffraction losses. This occurs at 11.7 GHz, i.e. for $q = 3$ and $m + 2n = 2$, and it is clearly visible in the simulated S_{11} from CST MWS, but a sharp dip appears in the Ansoft HFSS simulation (for the two associated modes, Q_d is of the order of 10^3). This difference probably comes from the inaccurate meshing in one of the softwares and it will be experimentally investigated afterwards. Another resonance condition is satisfied at 11.4 GHz, with $q = 2$ and $m+2n = 4$, however the three modes have large diffraction losses ($Q_d < 30$), therefore one finds a dip instead of a peak in the S_{11} spectrum.

Figure 3 shows the E-field in the nearly-confocal resonator with lateral open boundaries at 12 GHz obtained from CST MWS and Ansoft HFSS. The observed pattern obtained from the two softwares agrees very well with the theoretical expectations for the mode with $m = n = 0$ and $q = 4$: there are four nodes between the mirrors and the field has a Gaussian dependence on the radial distance r.

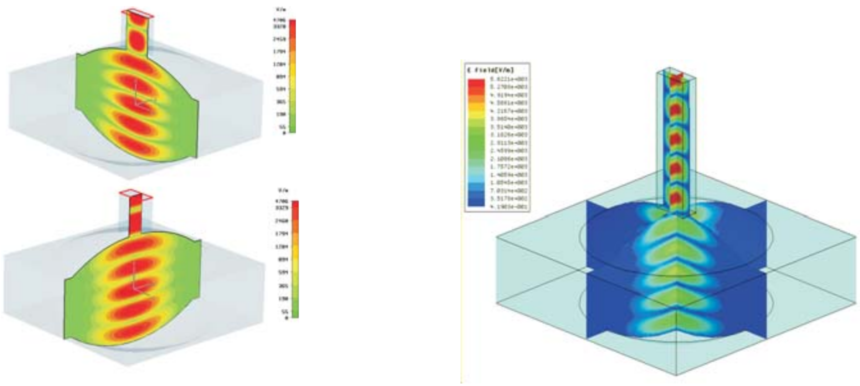

Figure 3: Eigen-mode found by CST MWS (left) and Ansoft HFSS (right) in the nearly-confocal resonator with lateral open boundaries at 12 GHz when a TE_{10} mode is injected through the waveguide.

3.2 Transmission coefficient for the incoming modes

Having successfully shown that the eigen-mode oscillating at 12 GHz inside the nearly-confocal resonator indeed has a large quality factor for the diffraction losses, the second step of our numerical studies is to demonstrate that the modes propagating inside a pipe do not couple to the nearly-confocal resonator at 12 GHz, as suggested by reciprocity.

The nearly-confocal resonator is installed on a rectangular pipe with a width of 11.0 cm and a height of 3.7 cm. The number of modes that can propagate through the pipe with a cut-off frequency below 12.1 GHz is 38 (24 TE modes and 14 TM modes). In the waveguide used for signal extraction, only the TE_{10} mode can propagate. As a result, one must compute 38 transmission coefficients around 12 GHz in order to fully characterize the coupling of the external TE and TM modes to the nearly-confocal resonator.

Around the frequency of interest, the transmission coefficient S_{21} was first simulated with a simple structure consisting of the rectangular pipe with the extraction waveguide only, i.e. with no nearly-confocal resonator structure. For some of the propagating TE and TM modes, the transmission coefficient S_{21} remains very small (-40 dB or less), because of their weak coupling to the TE_{10} mode of the extraction waveguide. Such modes are not relevant to the S_{21} simulations in the presence of the nearly-confocal resonator either. For all other TE and TM modes that are likely to have a reasonable coupling to the extraction waveguide, the presence of the two carved spherical mirrors in the upper or lower sides of the pipe leads to a dramatic decrease of S_{21} by more than 40 dB around the resonant frequency of the nearly-confocal resonator. This is illustrated in Figure 4 for the TE_{10} mode of the rectangular pipe. A similar behaviour is observed for all other incoming TE and TM modes. We have thus successfully demonstrated the possibility of using the nearly-confocal resonator to reduce the coupling to the external modes around the frequency of interest.

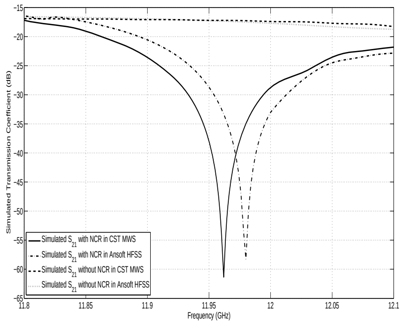

Figure 4: Simulated transmission coefficient of the TE_{10} mode between two ports: port1 (one end of the rectangular pipe) and port2 (the upper end of the waveguide).

4 Experimental results

In order to verify the validity of the simulations, a prototype was built according to the geometry of Figure 1. The length of the rectangular pipe is 30 cm. In addition, a 20 cm long standard M-band waveguide with transverse dimensions 1.905 cm × 0.953 cm is connected to the upper mirror through the plate, see Figure 5.

Figure 5: Photographs of the nearly-confocal resonator prototype installed onto a rectangular Aluminium pipe with a standard M-band waveguide connected to the upper mirror.

All the measurements were performed with a network analyzer (Agilent Technologies E8364B PNA series). The reflection coefficient of the nearly-confocal resonator was measured at the upper port of the waveguide. In order to get rid of the effect of the coaxial-to-waveguide adaptor, its insertion loss was measured as well.

Simulated S_{11} from CST MWS and Ansoft HFSS together with the measured result, are shown in Figure 6. Despite the absence of resistive losses in the simulations, good agreements between the numerical and experimental results are obtained. The simulated S_{11} slightly differs from the one shown in Figure 2, because of the presence of the Aluminium pipe, which affects the electromagnetic field that leaks out of the nearly-confocal resonator.

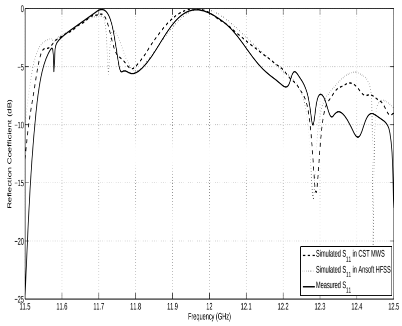

Figure 6: Simulated and measured reflection coefficient of the nearly-confocal resonator at the upper port of the waveguide.

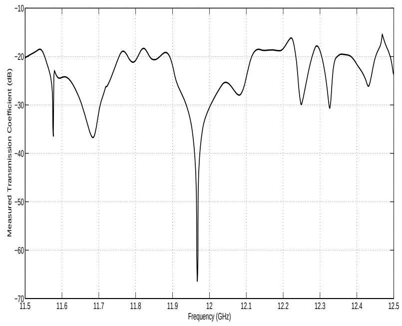

Figure 7: Measured transmission coefficient of the nearly-confocal resonator.

The transmission coefficient was experimentally studied as well, where a standard M-band horn antenna was employed to generate approximate TE_{10} mode electromagnetic waves at one end of the Aluminium pipe while the transmitted power was measured at the upper end of the waveguide. The waveguide and horn antenna were assumed to be lossless and the losses of the two coaxial-to-waveguide adaptors were subtracted. Figure 7 shows the measured S_{21}, where port 1 is the horn antenna output and port 2 is the junction between the upper mirror of the nearly-confocal resonator and the waveguide. As expected, a significant decrease of S_{21} is observed at the resonant frequency of the nearly-confocal resonator. The electromagnetic wave coming out from the horn antenna is mostly a TE_{10} mode, but some other modes may be present as well, especially if the horn is tilted. However, even in that case, a clear reduction of the coupling to external fields by the nearly-confocal resonator is observed (only the level of the off-resonance coupling is affected by the orientation of the horn with respect to the pipe axis).

Note the presence of a second dip at 11.7 GHz in the S_{21}, which corresponds to the resonance that has $q = 3$ and $m + 2n = 2$, though with a smaller quality factor for the diffraction losses than at 12 GHz, and thereby a larger coupling to the external modes.

5 Conclusion

The electromagnetic properties of a nearly-confocal resonator with spherical mirrors inserted onto an Aluminium pipe have been numerically and experimentally studied. As a result of the large quality factor associated to the diffraction losses, a dramatic reduction of the coupling to external sources and modes by the resonator was clearly achieved. In turn, this should allow a significant improvement of the signal-to-noise ratio for beam diagnostic devices based on such a nearly-confocal resonator configuration.

However, in-situ adjustments of the mirror spacing by remotely controlled actuators is likely to be mandatory for a proper operation of the device, as the dip around the resonant frequency is very narrow and as its position in the transmission coefficient depends strongly on the mirror distance. Also, damping material in the beam pipe, outside the resonator, can help reduce the influence of some parasitic TE and TM modes, but should meanwhile not affect the eigen-modes with a large Q_d value, since those are confined inside the resonator.

Acknowledgments

The authors wish to thank Lars-Erik Lindqvist at Uppsala University for the accurate manufacturing of the prototype.

This work is supported by the Commission of the European Communities under the 6^{th} Framework Programme "Structuring the European Research Area", contract number RIDS-011899.

References

[1] G.D. Boyd and J.P. Gordon, "Confocal multimode resonator for millimeter through optical wavelength masers," *Bell Syst. Tech. J.,* vol. 40, pp. 489-508, 1961.

[2] G.D. Boyd and J.P. Gordon, *Advances in Quantum Electronics*, J.R. Singer, Ed. New York, Columbia University Press, pp. 318-327, 1961.

[3] A. Battaglia, A. Gozzini and G. Boudouris, "Experimental study of confocal Fabry-Perot microwave resonators," *Nuovo Cim. S*, vol. 69, pp. 121-151, 1970.

[4] A.L. Cullen and A. Kumar, "The absolute determination of extinction cross-sections by the use of an open resonator," *Proc. R. Soc. Lond. A, Match. Phys. Sci.,* vol. 315, pp. 217-230, 1970.

[5] A. Fox and T. Li, "Resonant modes in a maser interferometer," *Bell Syst. Tech. J.,* vol. 40, pp. 453-488, 1961.

[6] H. Kogelnik and T. Li, “Laser Beams and Resonators,” *Appl. Opt.*,, vol. 5, pp. 1550-1966, 1966.

[7] N. Klein, “Proposal for a superconducting open resonator and its application as a gravitation balance detector,” Bergische Univ., Wuppertal, Germany, Ext. Rep., Feb. 1987.

[8] F. Caspers and G. Dome, “Reciprocity between pick-up and kicker structures including the far-field zone,” in *Proc. 4th Particle Accelerator Conf.,* London, U.K., 1994.

[9] G. Geschonke and A. Ghigo, “CTF3 design report,” CERN, Geneva, Switzerland, CERN/PS 2002-008 (RF), CTF3 Note 2002-047, LNF-02/008 (IR), YEAR.

[10] S. Cheng, A. Ferrari, M. Johnson, A. Rydberg, V. Ziemann and E. Öjefors, “Reduction of the coupling to external sources and modes of propagation by a nearly confocal resonator,” in *IEEE Trans. Micow. Theory Tech.,* vol. 55, Issue 10, 2007.

[11] http://www.cst.com/Content/Products/MWS/Overview.aspx

[12] http://www.ansoft.com/products/hf/hfss/